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Software Competence Center Hagenberg Softwarepark 21 A-4232 Hagenberg Tel. +43 7236 3343 800 Fax +43 7236 3343 888 www.scch.at Fuzzy Logic Laboratorium Linz Softwarepark 21 A-4232 Hagenberg Tel. +43 7236 3343 431 Fax +43 7236 3343 434 www.flll.jku.at

Program

13:00–15:00 Session 1 (Chair: Roland Richter)

| 13:00 | Frank Bauer: |
|-------|--|
| | On Theoretical Limits in Parallel Magnetic Resonance Imaging |
| 13:30 | Henrike Stephani: Dimensionality Reduction and Image Processing on Hyperspectral Terahertz Images |
| 14:00 | Wolfgang Heidl: Model-based analysis of gender-related variations in human decision-making |
| 14:30 | Edwin Lughofer: |

SparseFIS: Data-driven Learning of Fuzzy Systems with Sparsity Constraints

15:00 Coffee Break

15:15–17:15 Session 2 (Chair: Bernhard Moser)

| 15:15 | Bettina Heise: 2D-demodulation method applied for PS-OCT |
|-------|---|
| 15:45 | Thomas Klambauer: C++ Plugin Frameworks |
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On Theoretical Limits in Parallel Magnetic Resonance Imaging

Frank Bauer and Christian Clason

Abstract—Based on a Fourier series expression of true image, receiver sensitivities, and measurements, it is possible to give theoretical limits for the perfect reconstructability of image and sensitivities in parallel magnetic resonance imaging. These limits depend on the smoothness of the sensitivities, number of receiver coils, and size of the acquired k-space measurement window. Different types of a priori information can be incorporated in the determination of these limits. Furthermore, the method employed is constructive and can serve as the basis for a nonlinear reconstruction scheme, as is shown using data from a simulated phantom.

Index Terms—Magnetic resonance imaging, Image reconstruction, Fourier series, Newton method

I. INTRODUCTION

- AGNETIC Resonance Imaging (MRI) is a medical imaging method which employs radio pulse echoes to measure the hydrogen atom density, which allows the discrimination of different types of tissue. The spatial information is encoded, using a combination of gradient magnetic fields, in the phase and frequency of the timedependent echo, which is then measured by coils surrounding the patient. A Fourier transform of the recorded signal will therefore yield – line by line – an image of the investigated area (for a full discussion of the principles of MRI, see, e.g., [1], [2]). One of the major drawbacks of MRI in current practice is the speed of the image acquisition, since each line has to be acquired separately. The standard approach for speeding up the process acquires a subset of the lines (e.g., every second or every fourth). This, however, leads to aliasing, as the two-dimensional signal is now sampled below the Nyquist frequency (cf. Fig. 1 and the above references). As a remedy, Parallel Magnetic Resonance Imaging (PMRI) measures the radio echo using multiple complementary coils, which are usually placed in a circle around the patient. Since these coils have only limited aperture compared to a single coil, the resulting measurements are non-uniformly modulated. In this way one hopes to make up for the lost information.

Reconstruction strategies currently in use in daily clinical practice include SENSE [3], which is an algebraic linear least-squares recovery of the unaliased image using sensitivities taken from a fully sampled reference scan, and GRAPPA [4], where the missing Fourier coefficients are interpolated using an interpolation kernel fitted to additionally acquired lines around the zero frequency (in effect this is also fully sampled low-resolution reference scan). The quality of reconstruction therefore depends on the number of these so-called center lines, which reduces the speed-up gained from subsampling. Recently, nonlinear least squares methods for PMRI have been proposed [5], [6], [7], which treat the sensitivities as additional unknowns and were shown to allow improved reconstructions. Common to all algebraic reconstruction techniques is the danger of ghost respectively overfolding artifacts, which arise from incomplete separation of the superimposed parts in the aliased image (illustrated in Fig. 1d for a naive Tikhonov-regularized least squares solution using known sensitivities). In practice, of course, achievable reconstruction is also limited by physical constraints such as the signal-to-noise ratio (SNR), see, e.g., [8]. In the case of well known sensitivities there exists an extensive literature (e.g. [9], [10], [11]) giving account on the SNR of the reconstructions. In particular the noise imposed by measurement noise just depends mildly [9] on the undersampling.

In contrast to these works, we consider, as in GRAPPA and [7], the sensitivities as unknown and focus on the error imposed by the overfolding artifacts. We start by considering theoretical limits for the acceleration factor, in the sense that we derive algebraically necessary conditions under which it is in principle still possible, with a given number of receiver coils, to reconstruct image and sensitivities perfectly. It will be shown that this limit also depends in non-trivial ways on the smoothness of the sensitivities and the desired resolution. The main contribution of this work is thus to give guidelines for choosing appropriate set-ups for parallel imaging, which might be helpful both for SENSE and GRAPPA like methods. Additionally, the presented constructive approach can serve as the basis for the development of new reconstruction methods.

II. THEORY

A. Mathematical model and problem formulation

We consider the true image P as a function in $L^2([-1,1]^2)$. Since usually subsampling is only performed in one direction, we will treat the image as being composed of independent lines parallel to the aliasing direction,

F. Bauer is with the Fuzzy Logic Laboratorium, Johannes Kepler University of Linz, Softwarepark 21, 4232 Hagenberg, Austria (phone: ++43 7236 3343 432; fax: ++43 7236 3343 434; email: frank.bauer@jku.at). His work was supported by the Upper Austrian Technology and Research Promotion

C. Clason is with the Institute for Mathematics and Scientific Computing, University of Graz, Heinrichstrasse 36, A-8010 Graz, Austria, (email: christian.clason@uni-graz.at). His work was supported by the Austrian Science Fund (FWF) under grant SFB F32.



Fig. 1. Illustration of modulation and aliasing applied to a phantom: (a) True image, (b) modulated coil image, (c) modulated and sub-sampled (factor 2) coil image, (d) naive reconstruction suffering from ghost artifacts.

which can be expressed as a Fourier series

$$P(x) = \sum_{k=-\infty}^{\infty} p_k e^{ikx},$$

where we assume that at most $N_P + 1$ coefficients are nonzero. This allows us to write

$$P(x) = \sum_{k=-\lfloor N_P/2 \rfloor}^{\lfloor N_P/2 \rfloor} p_k e^{ikx}$$

where w.l.o.g. for odd N_P , we distribute the additional asymmetric coefficient to the positive part of the sum.

Similarly, let C be the number of receiver coils, with corresponding coil sensitivities $R^j \in L^2([-1,1]^2), j = 1, \ldots, C$, each of which can be written as

$$R^{j}(x) = \sum_{k=-\lceil N_{R}/2 \rceil}^{\lfloor N_{R}/2 \rfloor} r_{k}^{j} e^{ikx}, \qquad j = 1, \dots, C.$$

Again, $N_R + 1$ is the maximal number of non-zero coefficients for all sensitivities. We make the additional assumption that the sensitivities are much smoother than the image, i.e., that $N_R \ll N_P$, which in practice is always the case. Here, the excess coefficient for odd N_R is counted for the negative part of the sum.

A full measurement now consists of the Fourier coefficients of the point-wise multiplication of P and R_j . This is expressed as:

$$\sum_{k=-\lfloor N_P/2 \rfloor-\lceil N_R/2 \rceil}^{\lceil N_P/2 \rfloor+\lfloor N_R/2 \rfloor} m_k^j e^{ikx} = M^j(x) = R^j(x) \cdot P(x)$$
$$= \sum_{k=-\lfloor N_P/2 \rfloor}^{\lceil N_P/2 \rceil} \sum_{l=-\lceil N_R/2 \rceil}^{\lfloor N_R/2 \rfloor} p_l r_k^j e^{i(l+k)x}$$

Simple manipulation then yields

$$m_k^j = \sum_{\substack{a+b=k\\ -\lfloor N_P/2 \rfloor \le a \le \lceil N_P/2 \rceil\\ -\lceil N_R/2 \rceil \le b \le \lfloor N_R/2 \rfloor}} p_a r_b^j \tag{1}$$

In parallel MRI, only a subset of these coefficients are measured, which we denote by the index set $U \subset \{-|N_P/2| -$

 $\lceil N_R/2\rceil,\ldots,\lceil N_P/2\rceil+\lfloor N_R/2\rfloor\}$ which yields for $j=1\ldots,C$

$$\mathcal{S}_U\left(R^j(x)\cdot P(x)\right) = M_U^j(x)$$
$$= \sum_{k=-\lfloor N_F/2 \rfloor - \lceil N_R/2 \rceil}^{\lceil N_F/2 \rceil + \lfloor N_R/2 \rfloor} \delta_{k\in U} m_k^j e^{ikx}.$$

In standard applications we normally have an equidistant subsampling, i.e.

$$U = \Big\{ -\lfloor \frac{N_P}{2} \rfloor - \lceil \frac{N_R}{2} \rceil \le k \le \lceil \frac{N_P}{2} \rceil + \lfloor \frac{N_P}{2} \rfloor \Big| k/S \in \mathbb{Z} \Big\},$$

where S is the so-called acceleration factor. In all other cases we define $S := (N_P + N_R + 1)/|U|$.

Actually, an MR scanner physically determines exactly these coefficients m_k^j ; the sampling strategy (i.e., the decision which lines to measure) directly corresponds to a specific choice of U.

Since we are interested in theoretical limitations, we assume in the following that, regardless of subsampling, all $N_R + N_P + 1$ coefficients are acquired. Lower numbers (e.g., $N_P + 1$) can easily be treated by a modification of the index set U.

The problem we consider is now the following:

Problem 2.1: Given N_P, N_R and C, find the largest number S such that knowledge of

$$m_k^j, \qquad k \in U, \quad j = 1 \dots, C$$

can still determine uniquely

$$p_k$$
, $k = -\lfloor N_P/2 \rfloor, \ldots, \lfloor N_P/2 \rfloor$

and for $j = 1 \dots, C$

$$r_k^j, \qquad k = -\lceil N_R/2 \rceil, \dots, \lfloor N_R/2 \rfloor.$$

B. Minimum requirements for PMRI

Now we make use of the representation (1) to derive bounds on the acceleration factor S for which the resulting system of algebraic equations is uniquely solvable for p_k and r_k^j . To be precise, we derive necessary conditions which hold in the absence of any a priori information (like positivity or upper physical bounds) which might implicitly be used in specific algorithms. 1) Number of unknowns: A number of important MRI applications rely on image phase to provide critical information. A partial list includes proton resonance-based MR temperature mapping, phase-contrast velocity mapping for flow imaging in MR angiography, Dixon water/fat imaging, and phase-sensitive inversion recovery MRI (cf. [12]). Hence, in general, we have to assume that not only the coil sensitivities (and hence the measurements) but also the image is complex.

Thus, we need to reconstruct all N_c unknown coefficients,

$$N_c = N_P + CN_R + C + 1.$$

2) Available equations: By the given Fourier coefficients of the measurement, we have in total

$$N_E := C \cdot |U| = C \frac{N_P + N_R + 1}{S}$$

algebraic equations.

3) Algebraic conditions: According to the discussion above, we have in total N_E equations to determine N_c unknown coefficients. On the one hand, a system of npolynomials of degree m in n complex variables will have (with probability 1) exactly m^n different solutions [13]. On the other hand, a system of n + 1 polynomials in nvariables will in general have no solution. However, since we know that (in the absence of noise) the right hand side is by construction equal to the left hand side for the true image and sensitivities, we are in this special case always guaranteed the existence of at least one solution.

Therefore, a necessary condition for the unique solvability of this system of equations, and hence the reconstructability of P and R^{j} , is that the number of equations is strictly greater than the number of unknowns:

$$N_E > N_c.$$

This, of course, assumes that the receivers are linearly independent in the mathematical sense (for this, it is sufficient for the coil sensitivities to have complementary spatial variation, even if the receivers are not completely independent due to electromagnetic coupling and common noise). Due to the bilinear structure of the problem, adding a linearly dependent receiver does not contribute any additional information and hence will not increase N_E . In particular, this means that for the purpose of increasing the possible acceleration factor, at most $N_R + 1$ receivers can be used. (Naturally, more coils can be useful in other respects like noise reduction).

III. Results

If C, N_P, N_R are given, that means the highest possible acceleration factor S must satisfy

$$S < \frac{CN_P + CN_R + C}{N_P + CN_R + C + 1}$$

This allows us immediately to make the following observations:

- 1) Given arbitrarily fine resolution, i.e., $N_P \to \infty$, the maximal acceleration factor is equal to the number of coils, as expected. However, in any finite setting the maximal acceleration factor is *always* strictly smaller than the number of coils.
- 2) If N_P and C is fixed, S is determined by the smoothness of R^j . Specifically, the lower N_R , the larger the acceleration factor can be chosen. For twelve coils, $N_P = 256$ and $N_R = 16$ will theoretically allow S = 7, while the same N_P and $N_R = 128$ limits the acceleration factor to S = 2.

In general, if $N_R = \alpha N_P$ for some $0 < \alpha \leq 1$, we have

$$S_C(\alpha) = \frac{N_P C(1+\alpha) + C}{C\alpha N_P + N_P + C + 1}$$
(2)

Fig. 2a shows this function for different values of C and $N_P = 256$.

This behavior is understandable, since given the product $M = P \cdot R$, any factor of M having only N_R non-zero Fourier coefficients could be part of either P or R, and a unique recovery is not possible without additional constraints (such as adding additional independent measurements) or a priori information (such as an approximate measurement of the sensitivities).

Another interesting observation is, that adding more receivers does not give as much information as desired when the receivers are not very smooth (i.e. N_R is very small).

If, on the other hand, a specific acceleration factor S is desired, we can give a lower bound on the number of receiver coils necessary:

$$C > \frac{SN_P + S}{N_P + N_R + 1 - SN_R - S}.$$

Commonly, only $N_P + 1$ frequencies are acquired. In this case, we have the following limits:

$$S < \frac{CN_P + C}{N_P + CN_R + C + 1}$$

For arbitrarily fine image resolution, the maximal acceleration factor is still equal to the number of coils. For finite N_P , however, the achievable acceleration factor will be lower

$$S_C(\alpha) = \frac{N_P C + C}{C\alpha N_P + N_P + C + 1}$$

(in the example above, S = 6 and S = 1, respectively). Fig. 2b gives the maximal S in for various coil numbers and $N_P = 256$ for this situation. We see that we lose at most one acceleration factor compared to maximal acquisition.

The minimal number of receiver coils for a desired acceleration factor can be estimated by

$$C > \frac{SN_P + S}{N_P + 1 - SN_R - S}$$



Fig. 2. Maximal acceleration factor S depending on sensitivity smoothness $N_R = \alpha N_P$ for different coil numbers C and $N_P = 256$: (a) $N_P + N_R + 1$ lines are acquired, (b) $N_P + 1$ lines are acquired.

A. Incorporating a priori information

If one has further a priori information on the image or the sensitivities, it is possible to use this information to improve reconstructability. In our model, such information can be considered as additional equations.

1) Scaling invariance: The image and the sensitivities are only unique up to a constant factor, and one is in general only interested in relative, not absolute contrast in the image. Usually, this is treated by renormalization of the recovered image. Hence we can fix this factor by setting, e.g. $p_0 = 1$, which yields one additional equation.

This consideration also indicates that the bounds derived above are really just necessary conditions, as we cannot rule out that there are much more complicated hypersurfaces in the space of possible coefficients which all yield the same measurement and hence are not distinguishable.

2) Real images: If we can assume the image to be real (as, e.g., in standard MRI situations), we have additional information which gives us in total another $\lfloor N_P/2 \rfloor$ equations

$$p_{-k} = \overline{p_k}$$

for $0 \le k \le \lfloor N_P/2 \rfloor$ (in particular, p_0 is real). Now we have

$$N_E = \frac{C(N_P + N_R + 1)}{S} + \lfloor N_P/2 \rfloor$$

conditions, which lead to the bound

$$S < \frac{C(N_P + N_R + 1)}{\lceil N_P/2 \rceil + CN_R + C}.$$

- 1) Surprisingly, for $N_P \to \infty$, we have now that S can be equal to 2C.
- 2) The maximum achievable acceleration factor for finite resolutions is higher as well, but profits the

less the coarser the sensitivities are: e.g., S = 9 for $N_P = 256$ and $N_R = 16$, but still only S = 2 for $N_P = 256$ and $N_R = 128$. In general:

$$S_C(\alpha) = \frac{CN_P(1+\alpha) + C}{N_P(1/2 + C\alpha) + C},$$

the behavior of which is shown in Fig. 3a.

Again, acquiring only $N_P + 1$ instead of $N_P + N_R + 1$ k-space coefficients will at most decrease the maximum achievable acceleration factor by one.

3) Receiver normalization: In many MR scanners one tries to guarantee pointwise approximately the following identity:

$$\sum_{j=1}^{C} R^j = 1$$

which yields an additional set of $N_R + 1$ equations

$$\sum_{j=1}^{C} r_k^j = \delta_{0k}.$$

When one just tries to guarantee pointwise

$$\sum_{j=1}^{C} |R^j|^2 = 1$$

the underlying equations get a bit more complicated, however, this yields effectively just a set of another $N_R/2$ equations.

B. A numerical reconstruction scheme

The representation (1) can also be used as the basis for a numerical reconstruction scheme for parallel MR imaging, by considering (1) as a nonlinear system of equations

$$f(\mathcal{P},\mathcal{R}) = \mathcal{M}$$



Fig. 3. Maximal acceleration factor S depending on sensitivity smoothness $N_R = \alpha N_P$ for real images: (a) $N_P + N_R + 1$ lines are acquired, (b) $N_P + 1$ lines are acquired.

for the unknown $\mathcal{P} = (p_l)_l$ and $\mathcal{R} = (r_k^j)_{j,k}$ and given $\mathcal{M} = (m_k^j)_{j,k \in U}$. In matrix notation, this can be written as

$$f(\mathcal{P},\mathcal{R}) = \left((f(\mathcal{P},\mathcal{R}))_1, \dots, (f(\mathcal{P},\mathcal{R}))_C \right)^T$$

$$\begin{aligned} (f(\mathcal{P},\mathcal{R}))_{j} &= \\ \mathcal{S}_{\mathcal{U}} \begin{pmatrix} r^{j}_{-\lceil \frac{N_{R}}{2} \rceil} & & & \\ r^{j}_{-\lceil \frac{N_{R}}{2} \rceil + 1} & r^{j}_{-\lceil \frac{N_{R}}{2} \rceil} & & \\ \vdots & \vdots & \ddots & \vdots \\ & & & & r^{j}_{\lfloor \frac{N_{R}}{2} \rfloor} \end{pmatrix} \begin{pmatrix} p_{-\lfloor \frac{N_{P}}{2} \rfloor} \\ \vdots \\ p_{\lceil \frac{N_{P}}{2} \rceil} \end{pmatrix} \tag{3} \end{aligned}$$

or equivalently as

$$\begin{split} (f(\mathcal{P},\mathcal{R}))_j &= \\ \mathcal{S}_{\mathcal{U}} \begin{pmatrix} p_{-\lfloor \frac{N_P}{2} \rfloor} & & \\ p_{-\lfloor \frac{N_P}{2} \rfloor + 1} & p_{-\lfloor \frac{N_P}{2} \rfloor} & \\ \vdots & \vdots & \ddots & \vdots \\ & & & p_{\lceil \frac{N_P}{2} \rceil} \end{pmatrix} \begin{pmatrix} r^j_{-\lceil \frac{N_R}{2} \rceil} \\ \vdots \\ r^j_{\lfloor \frac{N_R}{2} \rfloor} \end{pmatrix}. \end{split}$$

where $S_{\mathcal{U}}$ is the subsampling operator as described beforehand (which removes rows corresponding to not measured coefficients).

Since we have more equations than unknowns, we want to solve this by an iteratively regularized Gauß-Newton method [14], [6], [7], i.e., by computing

$$\min_{(\delta \mathcal{P}, \delta \mathcal{R})} \frac{1}{2} \| f'(\mathcal{P}_n, \mathcal{R}_n) (\delta \mathcal{P}, \delta \mathcal{R})^T - \mathcal{M} + f(\mathcal{P}_n, \mathcal{R}_n) \|^2 + \frac{\alpha_k}{2} \| (\mathcal{P}_n, \mathcal{R}_n) + (\delta \mathcal{P}, \delta \mathcal{R}) - (\mathcal{P}_0, \mathcal{R}_0) \|^2$$

and setting $(\mathcal{P}_{n+1}, \mathcal{R}_{n+1}) := (\mathcal{P}_n, \mathcal{R}_n) + (\delta \mathcal{P}, \delta \mathcal{R}), \alpha_{k+1} := q\alpha_k$ with 0 < q < 1. The corresponding iteration can be

reformulated as

$$\begin{aligned} (\mathcal{P}_{n+1}, \mathcal{R}_{n+1}) &= (\mathcal{P}_0, \mathcal{R}_0) \\ &+ \left(f'(\mathcal{P}_n, \mathcal{R}_n)^* f'(\mathcal{P}_n, \mathcal{R}_n) + \alpha_n \right)^{-1} \\ f'(\mathcal{P}_n, \mathcal{R}_n)^* \left[\mathcal{M} - f(\mathcal{P}_n, \mathcal{R}_n) \\ &+ f'(\mathcal{P}_n, \mathcal{R}_n) ((\mathcal{P}_n, \mathcal{R}_n) - (\mathcal{P}_0, \mathcal{R}_0)) \right] \end{aligned}$$

which in practice is slightly faster to evaluate as well as more stable.

The derivative $f'(\mathcal{P}, \mathcal{R})$ acting on an increment $(\delta \mathcal{P}, \delta \mathcal{R})^T = (\mathcal{P}_{n+1} - \mathcal{P}_n, \mathcal{R}_{n+1} - \mathcal{R}_n)^T$ can be calculated explicitly using the product rule:

$$(f'(\mathcal{P},\mathcal{R})(\delta\mathcal{P},\delta\mathcal{R}))_{k,j} = \sum_{a+b=k} \delta p_a r_b^j + \sum_{a+b=k} p_a \delta r_b^j$$

or in brief

$$f'(\mathcal{P},\mathcal{R})(\delta\mathcal{P},\delta\mathcal{R}) = f(\mathcal{P},\delta\mathcal{R}) + f(\delta\mathcal{P},\mathcal{R}).$$

Hence, the derivative can be applied very quickly. Similarly, the evaluation of the adjoint derivative is rather straight forward and fast:

$$f'(\mathcal{P},\mathcal{R})^*(\delta\mathcal{M}) = \Big(\sum_{i=1}^C f^*(\mathcal{S}^*_{\mathcal{U}}\delta\mathcal{M}_i, R), \\ f^*(P, \mathcal{S}^*_{\mathcal{U}}\delta\mathcal{M}_1), \dots, f^*(\mathcal{P}, \mathcal{S}^*_{\mathcal{U}}\delta\mathcal{M}_C)\Big)^T,$$

where f^* is the multiplication with the Hermitian matrix corresponding to the appropriate matrix in (3) and $S^*_{\mathcal{U}}$ is the operator inserting zeros at non-measured parts.

Note that this method parallelizes trivially, since all lines along the fully sampled directions can be treated independently. Additionally, since only single lines are considered, the problem size is reduced significantly compared to a full image reconstruction: For standard image sizes, the iteration matrices fit inside the cache memory. In fact, due to the enormous parallelism inherent in this approach (hundreds of independent lines for a single slice, thousands for a full 3D data set) and the data locality, this method is well suited for implementation on graphics hardware. Another advantage of this approach over an image space based method is that the receiver sensitivities are determined by the measurements even in points x for which P(x) = 0 holds, due to the nonlocal coupling via the Fourier coefficients. On the other hand, it is possible to introduce coupling between neighboring lines as a penalty, which could be expected to give more stability in the presence of noise. Since this reduces the parallelizability, the trade-off should be considered in specific cases.

C. Numerical experiments

We show the feasibility of this approach by applying the proposed method for a single aliased line of the standard Shepp-Logan phantom (cf. Fig. 1). Setting $N_P = 256$ and $N_R = 10$ and measuring all $N_P + N_R + 1$ coefficients, the discussion in section II-B shows that using C = 2, 3, 4, 5 coils, the limiting acceleration factors are S = 1.9140, 2.7621, 3.5482, 4.2788 respectively—smaller factors should allow reconstruction, while larger factors should lead to failure.

We illustrate this using the one-dimensional Fourier transform of a single line from the Shepp-Logan phantom and random sensitivities (real and imaginary part normally distributed with mean zero and variance one). We start at a randomly perturbed image and sensitivity, and calculate new iterates using the IRGNM until the norm of the residual drops below 10^{-4} . The iteration converged in every case. We then plot the inverse Fourier transform of \mathcal{P} . Since the reconstruction can be unique only up to a constant, we rescale the transform such that the maximal value is one. The results are shown in Fig. 4, where the blue line shows the good reconstruction achievable with an acceleration factor chosen according to the calculations above, while the red line shows failed reconstructions for acceleration factors chosen too large.

To close this section, we show the reconstruction of a full 2D image using the described method. We generated the measurements sequentially from the (vertical) lines of a Shepp-Logan phantom with a resolution of $N_P = 128$. The receiver sensitivities (with $N_R = 6$) were independently and randomly generated for each of these lines and C = 12coils, and the resulting measurements subsampled with an acceleration factor of S = 4. As is usual in parallel MRI, we had to include center lines to achieve complete removal of aliasing artifacts if starting too far from the true solution (cf. Fig. 5a, which clearly shows aliasing artifacts). Here the central 3 coefficients sufficed. The result, reconstructed line by line, is shown in Fig. 5b, which is visually indistinguishable from the true image. To show the feasibility of parallelization, we started the computation of each line from the same constant initial guess $p_i = 0, r_i = 1$. Taking the previous line as the initial guess naturally leads to

faster convergence for each line. While this of course is still far from an actual reconstruction of parallel imaging data, it shows the promise of our approach for the development of a novel reconstruction method.

IV. DISCUSSION

In practical situations we face two major problems. On the one hand, the measurements are perturbed by noise, and on the other hand, although the sensitivities are very smooth, they still can have a rather large (most likely infinite) number of non-zero Fourier coefficients. Another problem which will occur in practice is that we actually do not know the number of relevant Fourier coefficients of the Receivers N_R exactly. This can lead to another kind of non-uniqueness: There might be two reconstructions with different receiver sizes which lead to exactly the same measurements, but which are completely different when looking at the image respectively the receiver coefficients.

However, a quick calculation using δ_k^j as the noise on the exact measurement m_k^j shows:

$$\overline{m_k^j} = \sum_{a+b=k} p_a r_b^j + \delta_k^j$$

and thus

$$\begin{split} |m_k^j - m_k^j| &\leq |\delta_k^j| + \|\mathcal{P}\| \|\mathcal{R}_j|_{N_R}\| \\ &+ \|\mathcal{P}|_{N_P}\| \|\mathcal{R}_j\| + \|\mathcal{P}|_{N_P}\| \|\mathcal{R}_j|_{N_R}\| \end{split}$$

where $\mathcal{R}_j|_{N_R}$ denotes the high-pass filtered version of \mathcal{R}_j without the first N_R Fourier coefficients, and similarly $\mathcal{P}|_{N_P}$ are the coefficients of the high-pass filtered image.

This means that as long as the amplitude of the higher Fourier coefficients is lower than the noise level, we can ignore these without further loss. Interestingly, this also means that noise limits the theoretical possibilities of undersampling perhaps in a severe way (cf. also [8]).

In practice, the dual issue of non-uniqueness and missing knowledge of the number of relevant Fourier coefficients could be dealt with by adding a sparsity constraint [15], which was not necessary in our test problems.

1) Center Lines: Indeed, the assumption that high frequency coefficients have lower amplitude than the central frequencies can be observed in virtually all realistic images and receiver sensitivities. In this case, the implicit assumption that all measured coefficients are of equal value, and that the sampling strategy can be completely specified by the number of measured coefficients, is no longer valid. Instead, it is then vital to acquire center lines, as can be explained using equation (1): If both the image and the sensitivity have very small Fourier coefficients outside the central (let's say 3) frequencies, only the terms p_1r_1 , p_1r_0 , p_0r_1 , p_0r_0 , and the corresponding terms involving the index -1 instead of 1 will be non-negligible. That implies that this information is only contained in the coefficients m_{-2} , m_{-1} , m_0 , m_1 and m_2 , although the coupling theoretically extends to more terms. So the omission of any of them due to the subsampling will lead to loss of uniqueness, even though the number of coils



Fig. 4. Reconstructions of one line of a phantom using the proposed method with different coil numbers and acceleration factors: (a) 2 coils, (b) 3 coils, (c) 4 coils, (d) 5 coils. Shown are the true image and two reconstructions from an acceleration factor which is smaller respectively larger than the limiting factor according to (2).

might be large enough (based on the theoretically possible coupling). It is therefore possible, using this relation and an estimate of the energy distribution of the frequencies of image and receiver, to determine the necessary center lines. On the other hand, the numerical method described above can be used for an efficient empirical determination of the required number of center lines.

2) Receivers: An important problem, which we have not understood in a quantitative way yet, is the interdependence between the stability of the proposed method and the receiver configuration. The investigations of this issue up to this point have all been of qualitative nature and yielded the expected results, i.e., that the less linearly dependent the receivers are, the better the method works.

However, the condition number of the matrix described in (3) seems to have a non-trivial dependence on the condition number of the matrix generated by the receivers, \mathcal{R} . Part of the cause might be that the solution of the whole problem is not unique up to (at least) a constant, and therefore ill-conditioning is always present, which dominates the (additional) ill-conditioning introduced by a bad choice of the receiver configuration.

It is important to remark that we have to face two opposing effects:

- The smoother the receiver sensitivities relative to the image, the higher the possible acceleration factor. This follows from equation (2) ff.
- The rougher the receiver sensitivities in absolute terms, the more receivers one can use (i.e. we have the possibility of higher acceleration factors), and the higher the SNR can be expected to be. This is due to the fact that a set of rougher sensitivities will more likely be linearly independent, and that the coupling between the coefficients of the image and



Fig. 5. Reconstruction of a 2D phantom with $N_P = 128$, $N_R = 6$, C = 12, S = 4: (a) no center lines, (b) 3 center lines.

the measurement (via the non-zero coefficients of the sensitivities) will be stronger (cf. IV-1).

A critical task in parallel imaging is therefore to balance these effects, for which our results can serve as a useful guide.

V. CONCLUSION

As seen in section III-C, this one-dimensional model together with the numerical method can replicate the features and problems inherent in the reconstruction problem of parallel imaging quite well. For this reason, it can serve as an effective tool in evaluating the feasibility of specific measurement configurations, as specified by the desired resolution, acceleration factor, the smoothness and number of coils, and especially the number of center lines, before committing to a full realization. A failure of the one-dimensional model, which constitutes a "best case" situation, will imply that other algorithms which do not specifically use extra assumptions will give poor and unreliable results as well.

Future work will be concerned with a quantitative estimation of the effect of center lines on the reconstructability of realistic data, and refining the numerical method for use on in vivo data. Furthermore, we expect that different sampling strategies such as radial sampling or subsampled 3D volume acquisition can be treated with a similar approach as well.

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Dimensionality Reduction and Image Processing on Hyperspectral Terahertz Images

Henrike Stephani Fraunhofer Institute for Industrial Mathematics (ITWM) Fraunhofer Platz 1 67663 Kaiserslautern Email: henrike.stephani@itwm.fraunhofer.de

Abstract—We propose a method for dimensionality reduction in hyperspectral Terahertz (THz) imaging based on Wavelet coefficients. The spectra are simulated in shape and characteristic peaks, a correlation analysis of the distances between the characteristic peak positions and the distances between the full and reduced dimensions is executed. The validity of the choice of reduced dimensions is demonstrated. Image analysis is performed on the remaining channels. A common foreground/background segmentation for all channels is calculated. On the foreground channel-wise smoothing is performed. The final visualization is achieved by performing hierarchical clustering on the smoothed channels. The result is an improved visualization in comparison to classical methods.

I. INTRODUCTION

hyperspectral imaging has been an evolving technique over the last years. It uses bandwidth other than the ones of visible light for image information retrieval. Usually those bandwidth are within the infrared and ultraviolet range [1]. Therefore, rather than having a monochromatic image with grey values or a color image, each pixel contains a vector with hundreds of values. Depending on the recording technique these values typically are time or frequency resolved. Instead of only having one two-dimensional image every data set is represented by a stack of as many two dimensional images as there are measured frequencies or time-intervals (called channels or slices). This fact confronts us with the problem of finding a good two or three dimensional representation of the data to make it visually interpretable. Different approaches are taken to do this. One is to show single features of the signals or spectra such as the maximal amplitude or time delay of each one. Another possibility often applied in infrared or THz spectroscopy is to use an integral norm. For example with Ω being the valid frequencies using the $L_1\text{-}\mathrm{Norm}$ for each spectrum $||f(\omega)||_1 = \sum_{\alpha} |f(\omega)|$. The problem with the former approach is that most $\overset{\Omega}{\text{of}}$ the information is lost. The problem with the latter that $||f(\omega)|| = ||g(\omega)||$ does not lead to the assumption that f is similar to g. Therefore, grouping the data with respect to such a norm is not advisable. A distance based approach should be used for that purpose. Here the data is grouped according to a distance function such that the distances between samples within one group are small and the distances between groups are big, this procedure is called classification. Both, supervised classification as well

as unsupervised classification also known as cluster analysis or clustering are used in hyperspectral imaging [2], [3], [4], we shall focus here on usupervised classification. To reduce negative effects in high dimensional data clustering, such as the curse of dimensionality , performing dimensionality reduction before clustering is an elementary task [5].

Another topic in hyperspectral imaging is the combination of spatial and spectral information. Image processing on that many dimensions is computationally expensive and sometimes difficult to define. Often spatial information is taken into consideration after the clustering, reassigning clusters depending on their neighborhood [4]. We shall use a different approach here and perform image processing on the channels or the reduced number of channels and thereby improve the actual clustering result. This is especially advisable if the data is very noisy. Otherwise the noise can accumulate over the multiple dimensions and have a dominating effect on the clustering result.

II. METHODS AND SIMULATION

The test data considered in this paper are hyperspectral pulsed THz measurements. These spectra are characterized by their broad and few peaks. For dimensionality reduction Wavelet coefficients shall be used. The different frequency level coefficients which the Wavelet transform provides are ideal if only certain broad features are of interest. Thus the typical broadness of the THz peaks is taken into consideration. To further enhance the visualization a number of image processing steps shall be preformed.

A. Dimensionality Reduction with Wavelet Coefficients

The high dimensionality of the information in hyperspectral imaging makes it hard to interpret the whole amount of image information. Hence, the main goal must be to reduce the dimensionality with as little information loss as possible. Terahertz spectra have some characteristics that lead to the presumption that this must be possible. Measurements of solids have - if any - few but broad peaks. Examples for these spectra can be seen in Fig. 1.

The Wavelet transform of a spectrum consists in the lossfree hierarchical decomposition with respect to certain basis functions. In the continuous case this decomposition has the



Fig. 1. Spectra of five different chemical compounds are presented, all have the named broad characteristics.

form $\gamma(s,\tau) = \int f(t)\psi_{s,\tau}^*(t)dt$ where $\psi_{s,\tau}(t) = \frac{1}{\sqrt{s}}\psi(\frac{t-\tau}{s})$ are the Wavelet basis functions generated by the mother Wavelet $\psi(t)$ by translation and scaling [6], [7]. The discrete Wavelet transform is calculated by down-sampling the amount of channels dyadically. In Fig. 2 this is illustrated. In this scheme the whole spectrum is iteratively filtered by the actual Wavelet functions $\phi(2^j t) \sum LP_{j+1}(k)\phi(2^{j+1}t - k)$ and a scaling function $\psi(2^j t) = \sum_{l}^{k} HP_{j+1}(k)\phi(2^{j+1}t - k)$.



Fig. 2. On every coarseness level j the frequencies are represented by the sum of Wavelet coefficients HP and scaling coefficients LP. The scaling functions are then further represented by coarser Wavelet and scaling coefficients and so on.

In viewing spectra with certain characteristic peaks, Wavelet coefficients of a coarser down-sampling level than the support of those peaks will not necessarily show these characteristics. At the same time coefficients of the level with down-sampling rate finer or similar to the given support must show them.

B. Simulating THz Spectra

The THz spectrum typically covers the frequencies from 100 GHz up to 2 THz while characteristic peaks have a width of at least 100 GHz. To determine weather just using coefficients of a certain decomposition level instead of the whole spectrum as well as to determine which range of Wavelet coefficients to use, we simulate the spectra. We start by simulating the



Fig. 3. Left hand side shows transmission of Aspirin and the reference measurement. Right hand side the transmittance.



Fig. 4. Basic shape of 10 simulated spectra, focus on a region with high variance on the right hand side.

coarse form of a transmittance spectrum, which is calculated from the transmission spectra of both the reference as well as the sample measurement. In Fig. 3 on the right hand side a typical example of the shape of these measurements can be seen. The presented spectrum is an Aspirin measurement. The characteristic peak at about 1 THz has a width of about 100 GHz. The transmission shown on the left hand side consists in the mere Fourier transform of a measured femtosecond pulse. The transmittance is the sample transmission divided by the reference transmission. It is customary to use the logarithmic spectra. The dynamic range implicated by the reference measurement typically covers frequencies from 100 GHz up to 3 THz. At the same time the sample measurement usually reaches the noise floor allready at 2 THz. The valid frequency range simulated here therefore shall cover 100 GHz up to 2 THz.

Because the goal of using the Wavelet transform is to detect the broad peaks of the spectra, the approach on simulation is focused on that as well. As the basic shape of the spectra we use the following function depending on x which takes values from 0.1 to 2, representing the THz range:

$$Y_{simulated} = \begin{cases} \exp(x^{-4+err}) & \text{if } x < 2, \\ (\frac{(x-2)}{2})^2 + y(2) & \text{else.} \end{cases}$$
(1)

In this equation *err* represents an error term for the gradient of the measurements, as different samples do usually not have the exact same shape. The error consist in a random variable of Gaussian distribution with expectation 0 and variance $\frac{1}{5}$, its effect can be seen in Fig. 4. We now simulate a number of one-peak-spectra as exemplified at the top of Fig. 5. The



Fig. 5. On top: Simulation of spectrum with only one peak situated around the same center of width 100 GHz. Bottom left: Wavelet coefficients 17 to 32, right: coefficients 33 to 64.

peak simulation is done using splines. The bottom level of Fig. 5 shows the Wavelet decomposition coefficients 17 to 32 and the coefficients from 33 to 64, i.e. the levels where j = 3 and j = 2. Both coefficients give a good impression of the position of the peak, though the finer coefficients at the bottom level give a more specific outline of the form of the peak, its existence is visible in both plots.

To further justify this impression by a quantitative analysis, systematic simulation of the spectra is done. We assume that in most applications a number of measurements of the same component is taken, e.g. in hyperspectral images we assume, that more than one pixel contains very similar spectral information. Therefore, over the valid frequency range we randomly distribute a number of centers and then use random values of Gaussian distribution with their expectation value at the respective center-points and variance 0.15. We then compare the distance between the thus produced points - i.e. the basis of the simulation - with the Euclidean distance between the chosen Wavelet coefficients. As a measure of quality the mean correlation between the distances is used. In mathematical terms:

Let c_i with $i \in \{1, ..., n\}$ be a random variable with $c_i \sim U[0.1, 2]$ and p_j with $j \in \{1, ..., m * n\}$ be a random variable with $p_j \sim N[c_{(j \mod n)}, 0.15]$. The distance matrix of these points $D(i, j) = d(p_i, p_j) = |p_i - p_j|$ is used as the basis for the quality measure. To this distance we compare the distance between the spectra. On the one hand over the whole frequency range, represented by D_{Sp} on the other hand over the Wavelet coefficients D_{Wa} . These distances between two spectra S_i and S_j are defined as follows:

$$D_{Sp_{ij}} = \sqrt{\sum_{x \in X} (S_i(x) - S_j(x))^2},$$

where $X = \{0.1, 0.1 + \frac{1.9}{N}, 0.1 + \frac{2*1.9}{N}, ..., 2\}$ and N is the



Fig. 6. Correlation of distance between points and distance between spectra and Wavelet coefficients respectively, depending on the number of centers chosen appearing in this simulation.

number of simulated channels.

$$D_{Wa_{ij}} = \sqrt{\sum_{k=17}^{32} (W_i(k) - W_j(k))^2}$$

As the measure of accuracy of the distance matrix D_{Sp} we compare it with the distance matrix of the points D by estimating Pearson's correlation coefficient between each two columns $D_{Sp}(:, S_j)$ and D(:, j) by:

$$C_{D_j D_{S p_j}} = \frac{\sum_{i \in I} (D_{(i,j)} - \overline{D_{(:,j)}}) (D_{S p_{(i,j)}} - \overline{D_{S p_{(:,j)}}})}{(m * n - 1)s_{(:,j)}s_{s p_{(:,j)}}}$$

where

$$s_{(Sp)_{(i,j)}} = \sqrt{n * m \sum_{i \in I} (D_{(Sp)_{(i,j)}})^2 - (\sum_{i \in I} D_{(Sp)_{(i,j)}})^2}$$

and $I = \{1, ..., m * n\}$. The accuracy measure then is defined as the mean correlation over all columns $\overline{C_{D_j D_{S_{p_j}}}}$. For the Wavelet coefficients $\overline{C_{D_j D_{S_{p_j}}}}$ is calculated analogously.

Wavelet coefficients $\overline{C_{D_j D_{Wa_j}}}$ is calculated analogously. One problem in performing the Wavelet transform and using only coefficients of a certain coarseness is that the values strongly depend on the starting point of the down-sampling window. If a peak appears closer to the edge of such a window the values are lower than if it is situated in the center of such a window. To reduce this effect we shift the windows by the mean number of points contained in a peak and use the mean of the respective coefficients as our features. As we use simulated spectra with about 256 points covering 100 GHz to 2 THz, i.e. 1.9 THz, the mean width of one peak would be 13 points. Therefore the Wavelet transform is performed from 13 different starting points.

Fig. 6 shows the plot of the above described correlations between the distances of the simulated spectra and the points chosen for the simulation. The x-Axis shows the number n of

centers c_i chosen. For each center m = 30 spectra are simulated that are situated around it. One can see that choosing only one center, the mean distance between the points has a higher correlation with the mean distance of the whole spectrum than the distance of the Wavelet coefficients. But as soon as spectra with different center locations are simulated this changes. This can be interpreted as small distances between peak-positions - as in the case of 30 spectra situated around the same center - being suppressed by just using a limited number of coefficients. At the same time bigger distances are enhanced, making the correlation between the down-sampled spectra and their peak position higher than between the original spectra and the peak position. The correlation was also calculated for the previous wavelet-levels (containing 64 and 128 wavelet coefficients), it was almost equal to the correlation with 32 coefficients, no improvement was achieved.

C. Image Processing

THz images - as they are measured pixel wise and the spatial resolution is low - show a variance in each pixels neighborhood. It is desirable to apply smoothing that considers the spatial and not only the spectral information of the image. Therefore, before the clustering, image smoothing will be performed. Otherwise the noise contained in each channel might be accumulated in the clustering and lead to wrong results.

The measurements usually show one or more objects which contain certain interesting features. While the different content of each object might only be visible in some channels, the objects themselves should be detected in every one of them. By using the common information of the given slices a first segmentation into foreground and background is done. This is achieved by channel-wise entropy based thresholding and binarization. The joint segmentation into foreground and background is done by assigning each pixel to the class the majority of the channels assigns it to. Thus we gain a mask that can be used to enhance the contrast between foreground and background and simplify the smoothing of the channels. Now an anisotropic diffusion filter is used on each two dimensional image. That means that the smoothing is applied according to the diffusion function:

$$\partial_t u = \operatorname{div}(g(|\nabla u|^2) \nabla u).$$

To influence the degree of edge preservation the diffusivity g is used. With constant g the diffusion would be linear and the image smoothed equally everywhere independently from the position of the respective pixel in the image. To avoid smoothing over edges the diffusivity usually is chosen in dependence of the gradient. Here the Perona Malik diffusivity is used [8]. It has the form:

$$g(|\nabla u|)^2 := \frac{1}{1 + |\nabla u|^2 / \lambda^2}.$$

D. Clustering

There is a broad variety of clustering algorithms, they are separated in at least two groups: partitional and hierarchical



Fig. 7. Left hand side shows a plastic figure, right hand side an envelope containing different objects.

ones. In partitional clustering the number of clusters needs to be known beforehand and often initializing parameters need to be chosen. Therefore although hierarchical algorithms are generally computationally more expensive they have the advantage of being more easily applicable on unknown data [9], [10]. In our case an agglomerative hierarchical clustering algorithm is used. It is initialized by assigning each sample point to an own cluster and then iteratively uniting the clusters closest together. The main influence parameters in hierarchical clustering are the distances between the samples and the distances between the clusters. The sample-vectors used consist of the calculated Wavelet coefficients calculated for each pixel spectrum. To determine the distance between each two, Euclidean distance is taken. This is sensible as an emphasis on high distances and a suppression of low distances is desirable. For the distance between the clusters the so called "complete" link function is used. When merging the clusters C_j and C_k the distance of the resulting cluster C_{jk} to another cluster C_l will be:

$$D(C_{jk}, C_l) = max(D(C_j, C_l), D(C_k, C_l)).$$

This link function is especially well applicable when edges between clusters are blurry and tend to blend into each other.

III. APPLICATION

The test data consists of two hyperspectral THz images. One of a plastic toy figure the other one of an envelope containing chemicals and highly absorbing material. THz spectroscopy and hyperspectral imaging are mainly based on pulsed THz systems where time domain signals (pulses) are recorded. From these pulses the transmittance is calculated by windowing, Fourier transformation, filtering, and division through a reference measurement. The transmittance of both measurements consists of about 300 channels from 0.1 up to 3 THz. The goal is to find a better visualization than the mere main amplitude plot that can be seen in Fig. 7. To achieve that, the above mentioned image processing steps are performed. The simulation showed that choosing 16 Wavelet coefficients as representatives for this spectrum already provides us with the main characteristic information.



Fig. 8. Masks of foreground and background segmentation calculated by thresholding of all slices.



Fig. 9. Coefficients 1,5,10, and 15. On the left hand side of each pair the unfiltered channel of the plastic toy figure measurement is shown, on the right hand side the smoothed and masked coefficient.

Therefore all following steps are only performed on these 16 channels. Fig. 8 shows the binarization of the images. As can be seen, by simple entropy maximization thresholding over all channels and morphological closing of the foreground, the resulting segmentation shows the shapes of the actual objects clearly.

The mask is then multiplied with the single slices and each two dimensional slice is smoothed with a diffusion filter. Some examples of this filtering can be seen in Fig. 9 and Fig. 10. Of both images coefficients that represent the low, medium, and high frequencies of the spectrum are presented. It can be seen that more differentiating features within the object are visible at the low frequency regions. The outline is clearer at the medium frequencies while at the high frequencies around 2 THz the outline of the objects becomes more and more blurry.

After image processing the clustering algorithm is applied to the data, the result is presented in Fig. 11. Both pictures



Fig. 10. Coefficients 1,5,10, and 15. The top shows the unfiltered channels of the envelope measurement, the bottom the smoothed and masked ones.



Fig. 11. Coefficients 1,5,10 and 15 of the Wavelet Masks of foreground and background segmentation calculated by thresholding of all slices.

show more information than the amplitude visualization in Fig. 7. In the plastic toy figure the different thicknesses of the object can be seen. Especially the core structure of the figure is detected. In the envelope the difference between the chemical - represented by the biggest object - and the other materials is detected by the clustering as well. This difference was not visible in the classical visualization.

IV. CONCLUSION AND FURTHER WORK

It was shown for simulated spectra of similar basic form and characteristics as THz transmittance spectra, that using a reduced number of Wavelet coefficients instead of the whole range of 256 channels simulated, is representative for the position of characteristic peaks within the spectrum. The quality measure applied for this was the correlation between the distances of the original position of the peaks and the distances between the respective coefficients. Using only 16 coefficients for a 2 THz spectrum lead to a slightly higher correlation with the peak position distances than using the original spectra. Using 32 or more improved the result even more. It can be concluded that Wavelets are an adequate method for dimensionality reduction in hyperspectral THz imaging. Further work should include analysis of the stability of this method towards noise and also the smoothing capacity of choosing these coefficients. Noise with higher frequencies than the characteristic peaks should automatically be smoothed by the choice of the coarseness level. The possibility of not choosing all coefficients of one coarseness level but only extreme values should also be further investigated.

Image analysis was performed to combine spatial and spectral information. It was executed in two steps: firstly by finding a foreground/background segmentation of the image and secondly by applying a diffusion filter on the thus masked channels. Afterwards clustering was performed. The clustering result presented an improved visualization of the hyperspectral image in comparison with usual techniques. Further possibilities for combining multi dimensional image information should be investigated. Furthermore the advantages and disadvantages of performing multidimensional smoothing as well as interchanging the single steps should be investigated, such as for example interchanging clustering and spatial smoothing [4].

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Model-Based Analysis of Gender-Related Variations in Human Decision-Making

Wolfgang Heidl

Profactor GmbH, Im Stadtgut A2, 4407 Steyr-Gleink, AUSTRIA

Abstract

Among manufacturing companies there is a widespread consensus that women are better suited to perform visual quality inspection, having higher endurance and making decisions with better reproducibility. We will utilize machine learning classifiers to model human decisions and analyze gender-related differences in this task. The analysis will be based on data gathered during thoroughly designed experiments with approximately 100 subjects.

1. Introduction

Quality control typically involves the visual inspection of products at the end of a production line. This task is quite often done exclusively by women. Their job is to make a quick good/bad decision and to sort out the bad products. Manufacturing companies often argue that women have more endurance in performing this task and also make decisions with better reproducibility.

This project aims at a mathematical description of decision-making processes in men and women during quality inspection tasks. It is focused on highlighting and quantifying the differences of male and female decision-making processes. The major goals of the project are:

- 1. Provide an answer to the question whether the gender-related differences in decision-making can be reproduced mathematically for visual inspection tasks.
- 2. Analyze the quantitative and structural nature of those variations in human decision-making.
- 3. Use the above results to guide development of machine learning classifiers specifically tailored to those applications.

WOLFGANG.HEIDL@PROFACTOR.AT



Figure 1. Preliminary sketch of the user interface presented during the experiments. The central part of the window will show the images the user classifies into good and bad. A queue of parts waiting for inspection is displayed to the left, stimulating subjects to maintain a preset average throughput.

2. Experiments

The analysis will be based on data gathered during thoroughly designed and controlled experiments with approximately 100 subjects, equally split between women and men. The subjects will be asked to perform quality inspection on synthetic images according to a predefined set of rules. The images will be created by samlping from a fault distribution \mathcal{D}_f . Each subject will be shown the same sequence of images to eliminate the influence of variations due to finite sampling from \mathcal{D}_f .

A preliminary sketch of the user interface presented during the experiments is depicted in Figure 1. While there will be no time limit on individual images, a certain overall throughput has to be reached. The required throughput and the complexity of the rules will be chosen such that typical subjects cannot stick to the rules exactly but need to devise simplifications. Conversly, the level of difficulty will be chosen such as to avoid degradation of the decisions into pure guessing.



Figure 2. Classifiers modeling human decisions in visual inspection. During training the classifier parameters (displayed as tuning knobs) are adjusted to minimize the discrepancy between human and classifier decisions.

3. Analysis

There is vast literature on modeling the problem of separating parts into good and bad in the fields of pattern recognition and machine learning. Various model architectures, so-called classifiers, have been devised along with suitable learning algorithms to identify the model parameters. However, these classifiers are mainly based on considerations of statistics and probability distributions of the features that they classify. The fact that the decision is originally made by a human is often neglected.

We will utilize and adapt classifiers to analyze the gender-related variations in human decision-making. On the input side, the classifiers will be fed with features vectors calculated from the images used during the experiment (Figure 2). The set of features will be chosen such as to represent the image characteristics relevant to the good/bad decision.

Classifiers create a decision boundary in feature space \mathcal{F} to separate good from bad samples. The shape and position of the boundary are controlled by the classifier parameters $\omega \in \mathcal{W}$ (Figure 3). During training these parameters and therefore the boundary are adjusted such as to minimize mis-classifications, i.e. samples laying on the wrong side of the boundary.

In fact, we seek to minimize the empirical risk of misclassifying new samples from \mathcal{D}_f by controlling classifier complexity to enforce proper generalization from the given samples. The resulting decision boundary and classifier parameters approximate a subject's decision behavior on the given fault distribution \mathcal{D}_f , not only on the samples presented during the experiments.

A key question for the analysis will be how to de-



Figure 3. Classifiers trained on different subjects. The decision boundary in feature space \mathcal{F} (depicted to the left) and corresponding parameters $\omega \in \mathcal{W}$ (right) of classifiers trained on indivudual subjects represent these subject's decision behavior. We hypothesize that the parameters representing male and femal subject's decisions are more similar within gender groups than across those groups.

fine an expressive distance measure between the parameters/structure of instances within one family of classifiers. Once such a distance measure is established, analyis can be performed in the induced space of subject-classifier parameters. We will then train a meta-classifier taking subject-classifier parameters as input and predicting the subjects's gender. The prediction accuracy of this classifier will be used to measure the correlation between subjects' gender and their decision behavior on images samples from \mathcal{D}_f . If the meta-classifier performs significantly better than random, the gender-related variation in visual quality inspection will be verified mathematically.

In order to analyze the nature of those variations we will resort to classifiers that yield interpretable structures such as decision trees, (fuzzy) rule systems, and nearest neighbor classifiers. To allow for analysis beyond computing the 'mean' female and male decision behavior, we will devise distance measures on classifier parameters that are in turn interpretable.

4. Classifiers tailored to human decisions

For the thrid project goal we will use the insights into human decision making from goal two to develop classifiers tailored to reproduce human decision boundaries for visual inspection tasks. At the core of these classifiers will be a grammar-like architecture similar to decision trees. Unlike decision trees we will allow for a broader spectrum of operations at the nodes, such as similarity-measures and trainable transformations, based on multiple variables of the input data.

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SparseFIS: Data-driven Learning of Fuzzy Systems with Sparsity Constraints

Edwin Lughofer¹*, Stefan Kindermann²

Abstract

In this paper, we are dealing with a novel data-driven learning method (SparseFIS) for Takagi-Sugeno fuzzy systems, extended by including rule weights. Our learning method consists of three phases: the first phase conducts a clustering process in the input/output feature space with iterative vector quantization and projects the obtained clusters onto one-dimensional axes to form the fuzzy sets (centers and widths) in the antecedent parts of the rules. Hereby, the number of clusters = rules is pre-defined and denotes a kind of upper bound on a reasonable granularity. The second phase optimize the rule weights in the fuzzy systems with respect to least squares error measure by applying a sparsity-constrained steepest descent optimization procedure. This is done in a coherent optimization procedure together with elicitation of consequent parameters. Depending on the sparsity threshold, more or less rules weights can be forced towards 0, switching off some rules. In this sense, a rule selection is achieved. The third phase estimates the linear consequent parameters by a regularized sparsity constrained optimization procedure for each rule separately (local learning approach). Regularization is necessary, as the learning problem might be ill-posed (in case of a singular covariance matrix to be inverted), leading to instable and incorrect results. Sparsity constraints are applied in order to force linear parameters to be 0, triggering a feature selection mechanism per rule. In some cases, this may also yield a global feature selection, whenever the linear parameters of some features in each rule are near 0. The method is evaluated based on high-dimensional data from industrial processes and based on benchmark data sets from the internet and compared to well-known batch training methods in terms of accuracy and complexity of the fuzzy systems.

Index Terms

Takagi-Sugeno fuzzy systems, iterative vector quantization, rule weight optimization, sparsity constraints, rule selection, feature selection

Corresponding author E. Lughofer¹* is with the Department of Knowledge-based Mathematical Systems, Johannes Kepler University of Linz, Austria, email: edwin.lughofer@jku.at

Stefan Kindermann² is with the Industrial Mathematics Institute, Johannes Kepler University of Linz, Austria, email: kindermann@indmath.uni-linz.ac.at

C++ Plugin Frameworks

Thomas Klambauer*

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Abstract

Continuing from the short introduction paper Generic image processing we delve into the further analysis of plugin frameworks, here FxEngine and LADSPA for C/C++, both with a focus on signal processing. A specific look is taken at the solutions to resource management and the overall architecture. Furthermore we present available tools for plugin interconnection and composition.

1 Introduction

In the quest for important plugin environment properties and solutions to common issues we analyze the frameworks of two products freely available. The first free as in "no cost", the second free as in "freedom" concerning open source software.

2 FxEngine Framework

The FxEngine Framework[1] is a closed-source C++ plugin framework licensed with an attribution clause and required author's permission for bundled distribution. It can be used on Linux and Windows platforms with DLLs and shared objects.

Signal processing is an important application area of the framework and the underlying concept of plugins reflects this fact, as they have multiple "Pins": for parameters and for the main input/output data. The architecture of the *engine* which connects and drives the plugins - which are here also called "Fx" - augments this impression of an signal processing system:



Figure 1: FxEngine architecture (FxEngine Doc)

^{*}Thomas@Klambauer.info

The principal different plugin categories and their interconnection in one Fx-Engine instance are shown in figure 1 on the preceding page with Renderer being a *sink*. Based on this network, arbitrarily large systems can be developed using the multiplicity of FxEngine objects illustrated in figure 2.



Figure 2: FxEngine multiplicity (FxEngine Doc)

FxEngine Framework uses an enum-based runtime type information for the media that is passed between plugins (by itself untyped). An interesting point hereby is, that multiple media types which are supported can be specified per pin. A wide variety of types in the field of audio and video as well as text are predefined and implementations of plugin types using that media are available.

Media data itself is intended to be transferred in data chunks allocated and managed by the IFxMedia class. The chunk size can be adjusted and filled with user-defined data, so that the passing of preallocated data is possible although it doesn't seem intended by the framework.

The basic work-flow is to instantiate an FxEngine, load plugins through its interface and then to establish *plugin interconnection* through that same interface. Then by running the engine the data processing starts. A visual editor, "FxEngineEditor" comes with the software package which performs this tasks and is able to store and load the network information in a proprietary file format. In figure 3 on the following page the connected input and output port of two plugins can be seen in the editor with "Hello World" as processed text data.



Figure 3: FxEngineEditor

Another interesting aspect of the Framework is the propagation of the processed data: the *synchronization of the plugins*. This can occur either by clock signals or a media request signal - a pushing versus pulling approach. The *error handling* and interface requesting bears strong similarity with the COM error handling of checking return values with macros and predefined error values.

Resource management, especially of memory shared between the plugins, which is another big concern for C++ module systems is managed by *allocation and release functions*. It can be observed, as with many other plugin and library systems that the problems with cross-module memory-management and possible inconsistencies with type definitions of objects crossing the boundary, due to the separate compilation of modules, drives or forces many developers into the direction of a C-style interface rather than an object-oriented one.

Amongst the available plugins also a LADSPA wrapper can be found enabling the integration of a wide variety of Audio plugins, which brings us to the next framework.

3 LADSPA

The "Linux Audio Developer's Simple Plugin API"[2], in short LADSPA, defines a simple C interface (ladspa.h) for audio-processing plugins under Linux. The framework only consists of this header file, which is licensed under LGPL[3]. It is settled in an open source environment, aimed at providing an VST - Virtual Studio Technology by Steinberg - replacement for Linux. A large number of plugins are available implementing this interface and a number of prominent hosts like "audacious" can make use of them. LADSPA does itself not introduce any dependencies.

A module can contain multiple plugins, whose functionality is accessed by requesting *descriptors*. Plugins have input/output *ports* for control and audio

data, which is passed simply as float arrays.

Plugins are controlled by functions specified by function pointers in the descriptor. See the listing below for the definitions.

```
1
   /* only export */
2
   const LADSPA_Descriptor * ladspa_descriptor(unsigned long Index);
3
   typedef struct _LADSPA_Descriptor {
4
     /* author info etc. */
\mathbf{5}
6
     /* port data */
7
8
     LADSPA_Handle (*instantiate)(const struct _LADSPA_Descriptor *
9
          Descriptor,
10
                                     unsigned long
                                         SampleRate);
^{11}
     void (*connect_port)(LADSPA_Handle Instance,
12
13
                             unsigned long Port,
14
                             LADSPA_Data * DataLocation);
15
     void (*activate)(LADSPA_Handle Instance);
16
17
     void (*run)(LADSPA_Handle Instance,
18
                  unsigned long SampleCount);
19
20
     void (*run_adding)(LADSPA_Handle Instance,
21
                          unsigned long SampleCount);
22
23
24
     void (*set_run_adding_gain)(LADSPA_Handle Instance,
                                    LADSPA_Data
25
                                                 Gain):
26
27
     void (*deactivate)(LADSPA_Handle Instance);
28
29
     void (*cleanup)(LADSPA_Handle Instance);
30
   } LADSPA_Descriptor;
31
```

An issue mentioned also in the documentation is the limited *error handling* capability. As solely the user's language C is assumed, no exceptions are expected or defined and no return values are designated to this. However this keeps the design very simple which is one of the stated goals. *Resource management* happens again with an allocation and release function pair.

For module interconnection, the LGPL'ed mixing software package GDAM -"Geoff & Dave's Audio Mixer" [4] contains functionality to chain LADSPA plugins visually by defining a flow graph. This mini-network can then be saved to XML whereof an encapsulating LADSPA plugin can be generated automatically, allowing for easy chaining and composing of plugins. In figure 4 on the next page an instance of this editor can be seen.



Figure 4: GDAM flow graph editor

4 Cross-Module Error Handling

In general, error handling in C++ is performed via *return values*, *exceptions* or a state information of some kind, where a module-level state and a function level result does not replace each other and can be complementary. Within modules applications and libraries alike - often exceptions are preferred, as they blend in naturally with the RAII pattern[5].

However with module boundaries, problems arise. As the C++ Standard does not concern itself with modules - they being an operating system concept - no ABI for them is specified. And also it is not specified how exception handling is implemented. Sutter and Alexandrescu in [6] even define as C++ coding standard rule #62: Don't allow exceptions to propagate across module boundaries.

This results from the fact the the exception handling code that is generated may be compiler/compiler-version and even compiler-flag dependent and can thus not be reliably guaranteed to the same across different modules.

With this in mind, it seems we best resort to error handling with return values.

5 Conclusion

We analyzed the FxEngine and LADSPA plugin environment with regards to *Resource Management, Module Interconnection, Error handling*, the interface and available tools in general. It was observed that error handling has a significant role and needs to be considered during plugin framework design. We also discussed *synchronization* and plugin composition as part of plugin frameworks, which may be required but the former introducing some additional complexity.

The work on plugin frameworks as part of "Generic Image Processing" will be continued in the same-titled bachelor thesis 1 .

References

- [1] http://www.smprocess.com.
- [2] http://www.ladspa.org/.
- [3] http://www.gnu.org/licenses/lgpl.html.
- [4] http://gdam.ffem.org/.
- [5] http://en.wikipedia.org/wiki/Resource_Acquisition_Is_ Initialization.
- [6] Herb Sutter and Andrei Alexandrescu. C++ Coding Standards. Addison-Wesley, 2004.

Nomenclature

| ABI | Application Binary Interface |
|--------|--|
| Fx | Plugin |
| Host | A plugin hosting environment (application) |
| LADSPA | Linux Audio Developer's Simple Plugin API |
| RAII | Resource Acquisition Is Initialization |
| VST | Virtual Studio Technology |

¹To be available at http://klambauer.info by August 2009

A Lightweight Model Driven Development Process based on XML Technology

Gábor Guta

Abstract: Model Driven Development and domain specific languages attract the attention of the industrial practitioners. Recently also more and more tools have become available to support these. Unfortunately these paradigms are typically discussed in the frame of the waterflow development process, which does not fit for small and mid sized agile teams.

To fill this gap, this article presents a lightweight, iterative, model driven software development process which was implemented and tested in industrial projects. After a short summary of the state-of-art of that field, we present the process in an abstract form. Then we give a detailed description of the actual realization based on XML technology. Finally we describe how the explained process and technology was applied in a real-world project.

Enhancing 3D Modelling Software for Inline Quality Control - Abstract

Alex Dănilă

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The purpose of this project is to:

- 1. Research and make a connection between established computer-aided design (CAD) workflows, geometric dimensioning and tolerancing (GD&T) and inline quality control.
- Conceive a set of tools for defining quality requirements on a CAD¹ model: CAD primitives, model relations, usage concepts and workflows.
- 3. Propose and possibly create these tools by extending an existing CAD system, BRL-CAD². This is the third part of the thesis, done as much as possible in the remaining time after doing the first two parts. The most important part of the work is the addition of unlimited custom properties to BRL-CAD primitives and sets of primitives, a functionality that all the studies open-source CAD programs is missing.

These tools are required for implementing generic inline quality control.

1 Introduction to Inline Quality Control

Inline quality control is the inspection that is done for all the instances of a component manufactured on the industrial line, rather than inspecting by sampling tests.

1.1 Failure rate, defect rate.

The complexity of the engineering products manufactured grows with time. The failure rate of a complex system is computed (with engineering assumptions) as the sum of failures of individual components³. Thus, the failure rate increases with complexity, and a lower failure rate of individual components must be obtained. The defect rate increases in a similar way to the failure rate, and

¹Computer-aided design, http://en.wikipedia.org/wiki/Computer-aided design

²http://brlcad.org/

³http://en.wikipedia.org/wiki/Failure_rate

as such the rate of defects in individual components must be small, or defect components should be found before assembly.

1.2 Quality control for 3D objects

The quality control of 3D objects is done by discovering the interesting properties of the objects using techniques like: using images to measure distances or find obvious defects (like cracks), doing a 3D reconstruction ⁴ (reconstruction is usually partial, because of occlusions).

When having the three-dimensional model of the real object, a series of automated verifications can be done: measure distances between the parts of the object and general tolerances, measure lengths and diameters, verify the quality of the surfaces (absence of holes and bumps, respecting of planarity or of other geometrical shape), measure quality of rigid joints, approximate volumes.

2 The connection between CAD, GD&T and inline quality control

CAD tools evolve into covering the entire product engineering process, this process being facilitated by: the increase in computing power, the advances in the computer vision field and the establishment of CAD standards, together with already established techniques in dimensioning and tolerancing for mechanical engineering projects.

The increase of the computing power of processors, together with the advances is computer vision, make the verification of more and more mechanical parts feasible.

Geometric dimensioning and tolerancing is an already established domain, although it lacks certain mathematical rigour. CAD formats and techniques have standardised and are being used in more of the production phases. By adding strong dimensioning and tolerancing capabilities to CAD programs, CAD software can move to also help with inline quality control

3 Conceive tools for defining quality requirements for CAD models

The following concepts will be defined during this project:

• Means of defining quality requirements by using primitives and other modelling techniques, based on existing CAD concepts. Also think of NURBS

 $^{^{4}\}mathrm{3D}$ reconstruction is the process of creating a three-dimensional model of an object from images of the object taken from different angles. More at http://en.wikipedia.org/wiki/3D reconstruction

surfaces and plus any other existing BRL-CAD primitives and modelling tools.

- Relations between primitives/model parts and properties for sets of primitives/model parts.
- Tolerance⁵ properties.
- Workflows and concepts as similar as possible to those already established in computer-aided design.

4 Propose means of extending BRL-CAD

The paper will propose a set of modifications for BRL-CAD, and implement a part of them in the remaining time.

- 1. Tolerance properties and means to store custom data along with BRL-CAD primitives, sets of primitives and objects. The open source 3D modelling software studied in the course of the project is missing this type of functionality.
- 2. The interface for adding the tolerances and specific properties.
- 3. Means of specifying quality properties other than by using tolerance values, using the existing BRL-CAD primitives and tools.
- 4. Primitives that are required for specifying quality properties that do not exist in BRL-CAD.

 $^{{}^{5}} Engineering \ tolerance: \ http://en.wikipedia.org/wiki/Engineering_tolerance$