Exercises Fuzzy Logic
November 11, 2010

9. Assume that $\lambda \in]-\infty, 0[$ and that $f : [0,1] \to [0,1]$ is defined by

   $$f(x) = e^{\frac{x^\lambda - 1}{\lambda}}.$$

   Show that f is a bijection. Is the function $T : [0,1] \times [0,1] \to [0,1]$ given by

   $$T(x,y) = f^{-1}(f(x) \cdot f(y))$$

   a t-norm?

10. Identify the t-norm T which is determined by the additive generator $t : [0,1] \to [0,\infty]$, $x \mapsto (1-x)^2$.

11. Identify the t-norm T which is determined by the additive generator $t : [0,1] \to [0,\infty]$, $x \mapsto \frac{1-x}{x}$.

12. Identify for each $\lambda \in]-\infty, \infty[$ the t-norm T_λ which is determined by the additive generator $t_\lambda : [0,1] \to [0,\infty]$, where t_λ is given by

    $$t_\lambda(x) = \begin{cases} -\log x & \text{if } \lambda = 0, \\ \frac{1-x^\lambda}{\lambda} & \text{otherwise.} \end{cases}$$

13. Show that the following implication holds: If $t : [0,1] \to [0,\infty]$ is an additive generator of a continuous Archimedean t-norm T and $c > 0$, then also $c \cdot t$ is an additive generator of T.