

# Fachwerk

## ■ Das Gleichungssystem

$$\begin{aligned} \text{glsystem1} = \left\{ f_1 + s_1 + 2 \frac{s_4}{\sqrt{5}} = 0, f_2 + \frac{s_4}{\sqrt{5}} = 0, -s_1 + s_2 = 0, -2f + s_5 = 0, \right. \\ \left. -s_2 + s_3 - 2 \frac{s_6}{\sqrt{5}} = 0, -f + s_7 + \frac{s_6}{\sqrt{5}} = 0, -s_3 - \frac{2s_8}{\sqrt{5}} = 0, f_3 + \frac{s_8}{\sqrt{5}} = 0, \right. \\ \left. f - 2 \frac{s_4}{\sqrt{5}} + 2 \frac{s_6}{\sqrt{5}} + s_9 = 0, -1 \frac{s_4}{\sqrt{5}} - s_5 - \frac{s_6}{\sqrt{5}} = 0, 2 \frac{s_8}{\sqrt{5}} - s_9 = 0, -2f - s_7 - \frac{s_8}{\sqrt{5}} = 0 \right\}; \end{aligned}$$

## ■ Vom Gleichungssystem zu Matrizen

? CoefficientArrays

CoefficientArrays[polys, vars] gives the arrays of coefficients of the variables vars in the polynomials polys.

```
CoefficientArrays[glsystem1, {s1, s2, s3, s4, s5, s6, s7, s8, s9, f1, f2, f3}] // Normal
{{0, 0, 0, -2 f, 0, -f, 0, 0, f, 0, 0, -2 f},
 {{1, 0, 0, 2/√5, 0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 1/√5, 0, 0, 0, 0, 0, 0, 0, 1, 0},
 {-1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, -1, 1, 0, 0, -2/√5, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 1/√5, 1, 0, 0, 0, 0, 0, 0},
 {0, 0, -1, 0, 0, 0, 0, 0, -2/√5, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 1/√5, 0, 0, 0, 1},
 {0, 0, 0, -2/√5, 0, 2/√5, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, -1/√5, -1, -1/√5, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 2/√5, -1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, -1, -1/√5, 0, 0, 0}}}
{b, M} =
CoefficientArrays[glsystem1, {s1, s2, s3, s4, s5, s6, s7, s8, s9, f1, f2, f3}]
{SparseArray[<4>, {12}], SparseArray[<27>, {12, 12}]}
```

## ■ Lösung

? LinearSolve

LinearSolve[m, b] finds an x which solves the matrix equation m.x == b.

LinearSolve[m] generates a LinearSolveFunction[...] which can be applied repeatedly to different b.

```
soll = LinearSolve[M, -b]
```

$$\left\{ \frac{16f}{3}, \frac{16f}{3}, \frac{17f}{3}, -\frac{13\sqrt{5}f}{6}, 2f, \frac{\sqrt{5}f}{6}, \frac{5f}{6}, -\frac{17\sqrt{5}f}{6}, -\frac{17f}{3}, -f, \frac{13f}{6}, \frac{17f}{6} \right\}$$

```
MatrixForm[{{s1, s2, s3, s4, s5, s6, s7, s8, s9, f1, f2, f3}, sol1}]
```

$$\begin{pmatrix} s1 & s2 & s3 & s4 & s5 & s6 & s7 & s8 & s9 & f1 & f2 & f3 \\ \frac{16f}{3} & \frac{16f}{3} & \frac{17f}{3} & -\frac{13\sqrt{5}f}{6} & 2f & \frac{\sqrt{5}f}{6} & \frac{5f}{6} & -\frac{17\sqrt{5}f}{6} & -\frac{17f}{3} & -f & \frac{13f}{6} & \frac{17f}{6} \end{pmatrix}$$

## Diskretisierung

```
glsystem2 =
```

```
{4 t1 == 50 + t2 + 10 + 10, 4 t2 == 50 + t3 + t7 + t1, 4 t3 == 50 + t4 + t8 + t2, 4 t4 == 50 + t5 + t9 + t3,
 4 t5 == 50 + t6 + t10 + t4, 4 t6 == 50 + 100 + t11 + t5, 4 t7 == t2 + t8 + 10 + 10,
 4 t8 == t3 + t7 + t12 + t9, 4 t9 == t4 + t8 + t13 + t10, 4 t10 == t5 + t11 + t14 + t9,
 4 t11 == t6 + t10 + t15 + 100, 4 t12 == t8 + 10 + 10 + t13, 4 t13 == t9 + t12 + t16 + t14,
 4 t14 == t10 + t13 + t17 + t15, 4 t15 == t11 + t14 + t18 + 100,
 4 t16 == t13 + 10 + 10 + t17, 4 t17 == t14 + t16 + t19 + t18, 4 t18 == t15 + t17 + t20 + 100,
 4 t19 == t17 + 10 + 10 + t20, 4 t20 == t18 + t19 + t21 + 100, 4 t21 == t20 + 10 + 10 + 100};
```

```
vars2 = Table[ToExpression["t" <> ToString[i]], {i, 1, 21}]
```

```
{t1, t2, t3, t4, t5, t6, t7, t8, t9, t10, t11, t12, t13, t14, t15, t16, t17, t18, t19, t20, t21}
```

```
{b2, M2} = CoefficientArrays[glsystem2, vars2]
```

```
{SparseArray[<15>, {21}], SparseArray[<81>, {21, 21}]}
```

```
LinearSolve[M2, -b2] // N
```

```
{26.1841, 34.7362, 41.0844, 47.6315, 56.2945, 70.4726,
 21.6765, 31.9697, 43.1472, 57.0738, 75.596, 21.9709, 35.9138,
 53.2575, 74.8374, 25.2797, 45.205, 70.4961, 31.7868, 61.9421, 45.4855}
```