Exercises Fuzzy Logic
November 25, 2010

14. Show that the following implication holds: If $t : [0,1] \to [0,\infty]$ is an additive generator of a continuous Archimedean t-norm T, if S is the dual t-conorm and if the function $s : [0,1] \to [0,\infty]$ is given by $s(x) = t(1-x)$, then for all $(x,y) \in [0,1]^2$

$$S(x,y) = s^{-1}(\min(s(1), s(x) + s(y)))$$

15. Let T be a continuous Archimedean t-norm and $\theta : [0,1] \to [0,1]$ a continuous, strictly increasing function with $\theta(1) = 1$, such that for all $(x,y) \in [0,1]^2$ we have:

$$T(x,y) = \theta^{-1}(\max(\theta(0), \theta(x) \cdot \theta(y))).$$

Is the function $t : [0,1] \to [0,\infty]$ given by $t(x) = -\log(\theta(x))$ an additive generator of T?

16. Determine the "implication" $\overrightarrow{T_{\mathbf{D}}}$ by evaluating the formula

$$\overrightarrow{T}(x,y) = \sup\{(u \in [0,1] \mid T(x,u) \le y\}$$

for the drastic product. Does in this case the following equivalence (residuum property) hold

$$u \le \overrightarrow{T}(x,y) \quad \Longleftrightarrow \quad T(x,u) \le y?$$

17. Show that for an R-implication $\overrightarrow{T}$ with respect to a left continuous t-norm T the assertion $\overrightarrow{T}(x,y) = 1$ holds if and only if $x \le y$.

18. Determine the negations using $N(x) = \overrightarrow{T}(x,0)$, which are induced by the minimum $T_{\mathbf{M}}$, by the product $T_{\mathbf{P}}$ and by the Łukasiewicz t-norm $T_{\mathbf{L}}$. Which properties do these negations have?

19. Use the negations from Example 18 to determine the disjunctions S corresponding to the respective t-norms T using the De Morgan formula

$$S(x,y) = N(T(N(x), N(y))).$$

Are these disjunctions always t-conorms?

20. Determine the R-implication induced by the nilpotent minimum as well as the corresponding negation and disjunction. Is the latter a t-conorm?