Surrogate-Based Multi-Objective Optimization of Electrical Machine Designs Facilitating Tolerance Analysis

Gerd Bramerdorfer[1], *Member, IEEE*, Alexandru-Ciprian Zăvoianu[2]

[1] Department of Electrical Drives and Power Electronics, Johannes Kepler University Linz, Linz 4040, Austria
[2] Department of Knowledge-Based Mathematical Systems, Johannes Kepler University Linz, Linz 4040, Austria

Multi-objective optimization algorithms are becoming ever more popular in the field of electrical machine design as they provide engineers with an automated way of efficiently exploring huge design spaces when searching for machines that are simultaneously highly competitive regarding several objectives like efficiency, material costs, torque ripple and others. Apart from exhibiting these good target characteristics, a good design should also be robust, i.e., it should not be very sensitive to slight changes in its design parameters as this would either seriously impact production costs or make the physical machine behave differently than its (optimized) computer simulation model. The present work is focused on describing how global surrogate models (i.e., nonlinear regression models), that are created in order to reduce the dependency on finite element (FE-)simulations during the multi-objective optimization run, can be easily reused to perform very fast local and global tolerance/sensitivity analyses of generated designs. While obtained in a fraction of the time required by the complementary FE-based approach, the surrogate-based sensitivity estimates are able to provide accurate and valuable information regarding the robustness of electrical machine designs. Ultimately, by integrating robustness-related information with Pareto front projections, we aim to provide engineers with much clearer pictures of the specific problem-related trade-offs discovered by the automated design optimization procedure.

***Index Terms*—cogging torque, efficiency, multi-objective optimization, optimal design, surrogate-based optimization, tolerance/sensitivity analysis, torque ripple**

I. Introduction

Engineers dealing with electric machine design usually try to find the best possible assembly for predefined objectives and constraints. For instance, the efficiency and the torque ripple are usually considered. In former times, the optimal design was obtained using analytical equations. Safety factors were typically introduced as the nonlinear effects could not be taken into account with satisfying accuracy. Subsequently, the use of finite element (FE-) simulations arose but the lack of computational power meant that the number of designs to be analyzed was severely limited.

As computing power has continuously increased, today, complex FE-simulations are typically run and nonlinear machine models are derived to quantify and compare the characteristics of machine designs. Moreover, a lot of design parameters with large parameter areas are investigated. Using optimization algorithms, an efficient exploration of the design space can be achieved and engineers have the option to perform very fine grained parameter discretizations or even optimize over continuous design spaces (whenever the latter makes sense from a technical/manufacturing point of view). Prominent examples of state-of-the-art optimization algorithms used in the field comprise the improved Strength Pareto Evolutionary Algorithm (SPEA2) [1], the Non-dominated Sorting Genetic Algorithm II (NSGA-II) [2], particle swarm optimization (PSO) [3], and various techniques based on differential evolution (DE) [4]. Furthermore hybrid or asynchronous algorithms have also been investigated [5], [6]. A typical optimization scenario can, e.g., be found in [7].

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Due to the economic pressure and the need for high power densities, cost-optimal machine designs receive preferential treatment. With this at the back of one's mind and applying the by now available techniques, highly-utilized machine designs are typically obtained. For instance, the ferromagnetic components are driven at very high flux densities. As a consequence, the obtained machine design most likely show an increased sensitivity regarding manufacturing imperfections or changes of the material characteristics. Such tolerances are unavoidable and thus researchers try to investigate their impact. For instance, the influence of the manufacturing technique on the material characteristics was treated in [8]–[11]. The results reveal that analyzing the sensitivity of particular assemblies with regard to tolerances is crucial. Further studies were carried out in [12]–[19]. However, usually just a single objective, an analytical model, or just a few design parameters are considered. Knowing the sensitivity of the obtained machine designs permits the engineer to trade off rated performance against robustness. Fig. 1 gives the explanation of robustness for a simplified single-objective problem (minimization) with a single design parameter. The design with the best overall rated performance as well as a less sensitive design are shown and the impact of tolerances for the design parameter x is illustrated.

Considering the reliability of machine designs with regard to parameter changes inevitably causes the computational cost to increase. Additional computations need to be conducted in order to derive predictions about the change of the performance with regard to tolerances. In particular, an accurate FE-based evaluation of the performances of machine designs incurs a

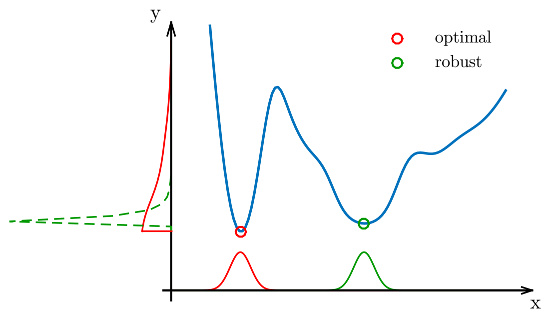

Fig. 1. Robust vs. optimal rated performance: While for rated operation the solution indicated by the red circle performs best, this is not necessarily the case when tolerances occur. An example for a Gaussian distribution of the selected design parameters and their impact on the output is presented.

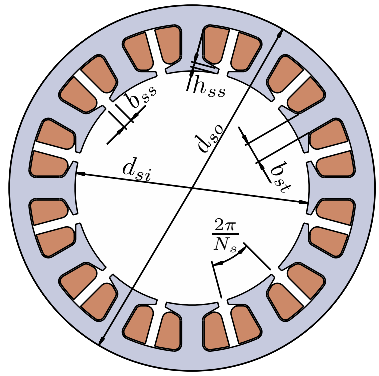

Fig. 2. Cross-section of the stator for the considered optimization problem.

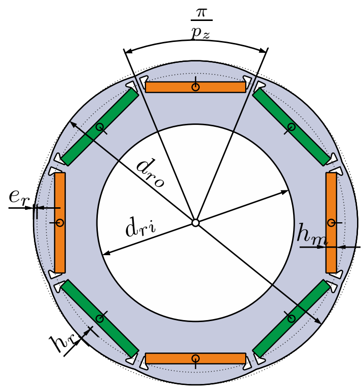

Fig. 3. Cross-section of the rotor for the considered optimization problem.

long runtime.

Some researchers already considered minimizing the computational cost for conventional optimization scenarios by generating surrogate models [20]–[24]. This allows replacing costly FE-simulations and speeding up the optimization process. This paper is about analyzing the feasibility of such surrogate models for conducting tolerance analyses. A typical optimization scenario for electric machine design is considered. Section II gives the experimental setup, discussing the design parameters and the objectives and providing a sketch of the motor topology to be investigated. As the electric machine features nonlinear characteristics, analyzing a design involves the run of multiple 2D-FE-calculations. While Section III explains a common FE-based approach for optimizing machine designs, Section IV illustrates the advantages when applying a surrogate-based strategy. For both approaches a computer cluster of 50 HTCondor™-managed [25] cores is used in order to analyze machine designs in parallel. In Section V, the applied sensitivity analysis strategy is described in detail. The derived measure of sensitiveness can be included as an objective for the optimization run. By contrast, Section VI highlights how a tolerance analysis could be incorporated to typical Pareto fronts of multi-objective optimization scenarios. This allows for identifying and differentiating between more sensitive and less sensitive designs. Thus, the user can manually select a machine design in due consideration of the effects caused by tolerances. We conclude by providing general remarks regarding the proposed technique and obtained results and by giving an outlook on future work.

II. Experimental Setup

The stator and the rotor for the considered optimization problem are presented in Figs. 2 and 3, respectively. Table I lists all the design parameters that are considered for optimization comprising the used designation, the symbol and unit, as well as the parameter range given by the minimum and maximum value and the step size. The total number of possible combinations is approximately 13.35×10^6. When having a closer look to the parameters it can easily be observed that most of them are related to the shape of the rotor. Even varying the stator inner diameter has a direct impact on the rotor, as the air gap width is kept constant. Further constant (geometry) parameters are presented in Table II. The installation space was fixed by defining the stack length and the stator outer diameter, while the change of the axial height of the end winding region was neglected. Besides the temperature of the permanent magnets and the stator coils, the considered load point, defined by rated torque and speed, is also denoted in this table.

The scenario comprises multiple objectives, as they are the material costs of the assembly, the cogging torque at no load operation and the machine's efficiency and the torque ripple for the given load point. Table III lists the four objectives. The material cost c_{mat} involve the cost for coils made of copper c_{coils}, the cost for the Neodymium-Iron-Boron (NdFeB-) mag-

TABLE I
Considered Design Parameters

Name	Symbol / Unit	Min.	Step	Max.
magnet height	h_m [mm]	1.0	0.2	4.0
magnet pole pitch	α_m [1]	0.70	0.05	1.00
pole height	h_r [mm]	0.5	0.1	2.0
pole eccentricity	e_r [mm]	0.0	0.1	2.5
stator inner diameter	d_{si} [mm]	45.0	0.5	50.0
stator tooth width	b_{st} [mm]	3.5	0.1	6.0

TABLE II
CONSTANT AND RATED PARAMETERS

Name	Symbol [Unit]	Value
number of stator slots	N_s [1]	12
number of rotor poles	p_z [1]	8
air gap width	δ [mm]	0.8
slot width	b_{ss} [mm]	2
stator outer diameter	d_{so} [mm]	75
rotor inner diameter	d_{ri} [mm]	28
stack length	l_{stack} [mm]	40
coils' temperature	ϑ_{coil} [°C]	120
permanent magnets' temperature	ϑ_{pm} [°C]	90
rated torque	T_{rate} [Nm]	0.5
rated speed	n_{rate} [rpm]	3000

TABLE III
OBJECTIVES

Name	Symbol / Unit
material costs	c_{mat} [Euro]
peak-to-peak cogging torque	T_{cogpp} [Nm]
efficiency	η [%]
peak-to-peak torque ripple at load	T_{rippp} [Nm]

nets c_{magnet} and the cost for the core material c_{core}. As the assembly is modeled using common CAD-software, the total volumes of the permanent magnets v_{magnet} and the coils v_{coils} can be easily obtained. The volume of the stator core and rotor core can be similarly derived. However, when considering the waste during manufacturing, the required volume v_{core} is usually defined using a quadratic cross section with side length of the stator outer diameter d_{so}:

$$v_{core} = d_{so}^2 \, l_{stack} \; . \tag{1}$$

Thus, the cost for the core material are constant for all possible design candidates. Using the mass density of the considered components, the masses can be obtained. This finally allows to calculate the material costs, as usually the prices are given per kilogram. Here, the defined price of the magnets is 50 €/kg, while 8 €/kg and 2 €/kg are defined for copper and laminated steel of grade M400-50A, respectively.

Typically, the material cost and the efficiency are conflicting objectives. Thus, when analyzing the design space, no unique assembly featuring superior characteristics can be identified. Instead, the solution of the optimization problem comes in the form of a set of Pareto non-dominated designs (notation: PN). Each element (i.e., design) from the PN has the property that it is not worse than any other element in the set with respect to all four objectives under considerations – i.e., no element in this set is fully Pareto dominated by another element from the set. The projection of the PN in objective space is called the Pareto front (notation: PF).

Finally, the decision maker chooses (subjectively) a design from the PN by deciding what is the suitable trade-off between the objectives for the goal at hand. The general task of the optimization strategy is to present the decision maker with an accurate PN that (objectively) models all the existing trade-off as close to perfection as possible.

III. COMMON OPTIMIZATION STRATEGY

In order to derive accurate results for the objectives for any machine design under consideration, the analysis of the machine performance is done by applying two-dimensional nonlinear magnetostatic finite element (FE-) calculations.

The machine designs under consideration feature a rotor with buried magnets (see Fig. 3). Thus, a considerable torque due to the rotor saliency can be generated. From this it follows that the optimal current angle must be separately derived for any design under consideration. To allow for deriving this parameter for maximum motor efficiency and to further compute the torque ripple for the respective current vector, a nonlinear modeling of the machine characteristics must be considered. The data required for modeling is obtained by running multiple FE-simulations with different current vectors per machine design under consideration. Successfully implemented techniques can be found in different articles, e.g., [26]–[32].

As a single machine design thus requires a considerable time for the analysis of its performance (i.e. several minutes when using a computer cluster and analyzing FE-simulations in parallel) and the total number of possible machine designs is very high, a grid search cannot be completed within satisfying computation time. Thus, many researchers started to experiment with advanced search/optimization techniques from the field of soft computing that are able to explore the design space more efficiently. Suitable choices include, e.g., genetic algorithms [1], [2], particle swarm optimization [3] or differential evolution [4].

Multi-objective evolutionary algorithms (MOEAs) [33] have proven especially successful because of their inherent ability to discover sets of nearly Pareto-optimal machine designs during single runs. Thus, it is possible to obtain high-quality designs without the need to evaluate all possible parameter combinations. Nevertheless, since all the aforementioned optimization techniques are population-based (see Fig. 4), they still require a relatively large number of designs to be evaluated during a single optimization run. Usually, several hundred to a few thousand designs need to be explored during a single MOEA run. When also factoring the dependency on FE-simulations, one can argue that simply applying MOEAs makes the optimal design problem viable but by no means fast to solve – e.g., a complex MOEA-based optimization of an electrical machine design distributed over a computer cluster can still take nearly a week [5].

Because of the complex interactions involved (CAD-systems, FE-based simulations, various optimization techniques/MOEAs, etc.), solving optimization problems concerning electrical machine design usually necessitates an appropriate software framework. For this analysis, MagOpt [34], [35] is used. This software allows for automatically analyzing and solving multi-objective optimization problems and features a flexible analysis structure. Usually it is applied when dealing with scenarios where the evaluation of design candidates requires high computational cost. A particular optimization problem solved with MagOpt was for instance presented in [7]. Many different techniques for minimizing the runtime of optimization problems were developed and implemented to MagOpt in the past. Among others, evolutionary algorithms [1], [2] and surrogate modeling techniques [20], [36] were considered.

MagOpt is developed at both, the Linz Center of Mechatronics [37] and the Department of Electrical Drives and Power Electronics at the Johannes Kepler University Linz, in Linz, Austria.

IV. Surrogate-based Optimization

As we wish to improve the very long run times required by MOEA-based optimizations, we adopt a hybrid optimization strategy (initially outlined in [20]) that uses global surrogate models automatically created on the fly (i.e., during each optimization run).

These surrogates are (ensembles of) linear and nonlinear regression models that are trained to predict target (i.e., objective and constraint) values based on given inputs (i.e., arrays of design parameters). Speed improvements are obtained by switching the MOEA to a surrogate-based fitness assessment function – that has no CAD and FE dependencies – during the middle part of the optimization runs. After evaluating several candidates, multi-layer perceptrons [38] (MLPs) – a specific type of artificial neural networks – have proven to display the best accuracy vs. training time trade-off for the nonlinear targets we want to predict.

Further improvements of the total run time can be obtained by using ensemble-based surrogates trained over Pareto-trimmed training sets [21]. Specifically, for each optimization target, the ensemble surrogates simply average the predictions of the 10 best individual MLP models obtained after a predefined limited best parameter grid search. These fast-to-train predictors also allow for the integration of multiple *surrogate-based optimization blocks* inside an optimization run with the hope of exploring a bigger part of the search space. An overview of the resulting surrogate-assisted optimization procedure is presented in Fig. 5.

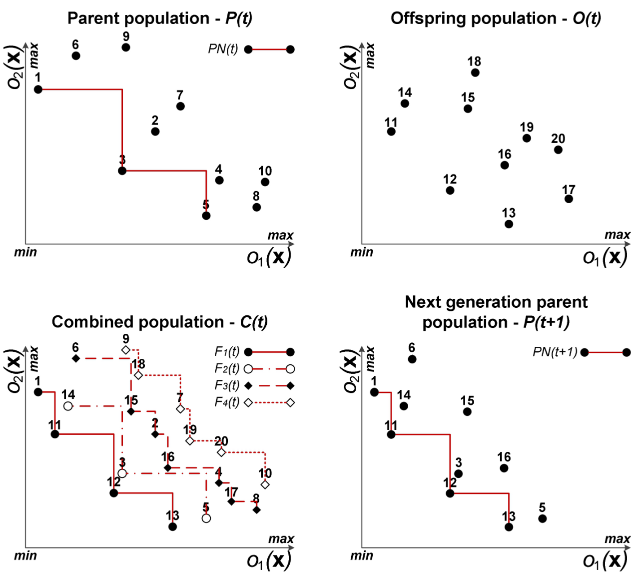

Fig. 4. Conventional optimization strategy - for instance, the multi-objective minimization problem is solved by applying NSGA-II, a very well-known multi-objective evolutionary algorithm that relies on a strategy intended to generation-wise move its population towards the Pareto-optimal set.

One important aim of the present work is to show how the global surrogate models created to speed up the MOEA-based optimization runs can also be used to perform a very fast but accurate tolerance analysis of the resulting optimized electrical drive designs.

V. Sensitivity analysis

In [39], Creveling describes in depth several tolerance analysis techniques. In the present work we focus on the most common off-line sensitivity/tolerance analysis strategy: the min-max approach. Given a design parameter vector $\mathbf{x} = \{x_1, x_2, \ldots, x_n\}$ and m objectives and constraints (i.e., targets) to analyze $\mathbf{y}(\mathbf{x}) = \{y_1(\mathbf{x}), y_2(\mathbf{x}), \ldots, y_m(\mathbf{x})\}$, we:

- supply the allowed deviation/tolerance intervals for each of the n variables either using a relative formulation (e.g., $\pm p_c\%$ of the nominal value of variable x_c) or a nominal-value formulation (e.g., a lower-bound and upper-bound interval $[l_c, u_c]$ with $l_c \leq 0$, $u_c \geq 0$ and $l_c \neq u_c$ that is centered around x_c);
- determine for $i \in 1, \ldots, m$ an accurate estimation for y_i^{min} and y_i^{max} – the expected min and max values of target y_i when the n variables can take any value within their allowed deviation intervals;
- compute informative metrics that show the magnitude of the expected target changes;

In order to evaluate the precision of our surrogate-based tolerance/sensitivity analysis approach, we double-checked the obtained results with FE simulations. Thus, because of the limited number of evaluations that can be performed during a (time-constrained) FE-based tolerance analysis, the general strategy we propose has two distinct stages:

- a primary estimation of sensitivity concerning *local changes* that affect a single design variable, i.e., the standard one-factor-at-a-time (OFAT) [40] approach;
- a secondary estimation of sensitivity concerning *global interactions* between all the considered design variables.

The purpose of the (optimistic) OFAT approach is to provide an accurate picture of the individual impact induced by each design variable on each elicited target. The more computationally-intensive and more imprecise (stochastic) global sensitivity estimation aims to discover if interactions between the influences exhibited by different design variables are likely to amplify the min-max estimates obtained via OFAT.

A. Estimation of Local Sensitivity

For a design variable vector $\mathbf{x}$ of size n, the local sensitivity of target $y_i(\mathbf{x})$ due to the sole variation of the design variable x_c, $c \in \{1, \ldots, n\}$ inside the interval $[x_c + l_c, x_c + u_c]$, where $l_c \leq 0$, $u_c \geq 0$ and $l_c \neq u_c$, is estimated by:

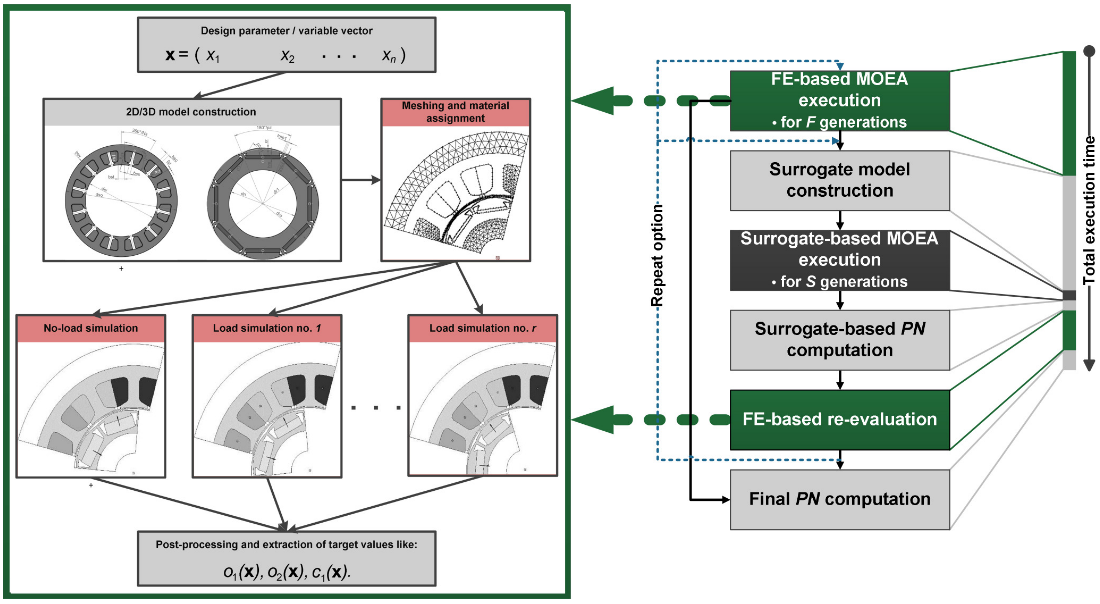

Fig. 5. An overview of the surrogate-based optimization process used in the present research. There are three hypothetical targets that must be considered: two optimization objectives (o_1 and o_2) and one constraint (c_1).

- evaluating 11 different design variations, $\mathbf{x^0}, \mathbf{x^1}, \ldots, \mathbf{x^{10}}$ where:

$$x_j^k = \begin{cases} x_j + l_c + 0.1 \cdot (u_c - l_c) \cdot k, & \text{if } j = c \\ x_j, & \text{if } j \neq c \end{cases}, \quad (2)$$

with $k \in \{0, 1, \ldots, 10\}$ and $1 \leq j \leq n$;
- selecting the two extreme target values y_i^{min} and y_i^{max} observed in the aforementioned 11 design variations;
- computing the *relative (elasticity) indicator*:

$$\Delta_{loc}(y_i, x_c, l_c, u_c) = \frac{x_c \cdot (y_i^{max} - y_i^{min})}{y_i \cdot (u_c - l_c)} \quad . \quad (3)$$

Eq. (2) means that, in order to obtain the 11 design variations associated with the design variable x_c, we assign to x_c 11 uniformly sampled values from its predefined variation interval $[x_c + l_c, x_c + u_c]$ while fixing the values of all other variables from the design vector we wish to analyze. If a relative formulation is preferred, l_c and u_c can be determined instantly as $l_c = -\frac{x_c \cdot p_c}{100}$ and $u_c = \frac{x_c \cdot p_c}{100}$ when one considers a design tolerance range of $\pm p_c\%$ of the nominal value of variable x_c.

Since this design vector is of size n, and each variable needs to be varied separately, a total of $11 \cdot n$ design variations must be analyzed during the OFAT local sensitivity analysis of each target.

Eq. (3) provides a good estimation of the relative influence of variable x_c on target y_i (when assuming local linearity). Δ_{loc} should be interpreted as follows: for a change of $p_c\%$ in x_c, one should expect a change of $p_c \cdot \Delta_{loc}(y_i, x_c, l_c, u_c)\%$ in y_i. The relative formulation allows for an easy comparison between the OFAT sensitivities induced by a variable across several targets or between the OFAT sensitivities induced by all variables on a single target.

B. Estimation of Global Sensitivity

The global sensitivity of target value $y_i(\mathbf{x})$ with regard to the simultaneous variation of all the design variables of $\mathbf{x}$ inside predefined deviation intervals is estimated by analyzing an extra 1000 "neighboring" designs. The procedure that generates these 1000 design variations depends on n, the number of design variables in $\mathbf{x}$. Thus, if $1 < n \leq 9$:

- 2^n individuals will be created by performing a grid combination of extreme values. The reasoning is that under the assumption of local linearity, extreme target values correspond to extreme variable values (i.e., to $x_c + l_c$ or $x_c + u_c$). Accordingly, stronger interactions are also expected between combinations of such variables (e.g., in design vectors like $\mathbf{x}^* = (x_1 + l_1, x_2 + l_2, \ldots, x_n + u_n)$).
- $1000 - 2^n$ individuals will be created using a Latin Hypercube sampling (LHS) procedure [41].

If $n > 9$, all 1000 individuals are generated via LHS, thus yielding in a slightly more unstable (fully stochastic) global sensitivity estimation.

After generating the required "neighboring" designs, the global sensitivity of target value $y_i(\mathbf{x})$ is computed by:

- evaluating (i.e., estimating the target values of) the 1000 extra design variable vectors;
- selecting the two extreme target values y_i^{min} and y_i^{max} among the $1000 + 11 \cdot n$ total (global+local) number of available design variations;
- computing

$$\Delta_{glob}(y_i) = \frac{x_{spr} \cdot (y_i^{max} - y_i^{min})}{y_i \cdot [x_{spr}^{max} - x_{spr}^{min}]} \quad (4)$$

where x_{spr} is the base value of the design variable that displays the largest relative deviation when looking at the variable vectors associated with y_i^{min} and y_i^{max}. x_{spr}^{max}

and x_{spr}^{min} mark the min/max values of x_{spr} inside the variable vectors associated with y_i^{min} and y_i^{max}.

$\Delta_{glob}(y_i)$ shows the relative expected change of target value $y_i(\mathbf{x})$ when given the possible variation of all the design variables of $\mathbf{x}$ inside their predefined tolerance intervals.

A small example is useful in order to better illustrate how x_{spr}, x_{spr}^{max} and x_{spr}^{min} are obtained. Let us consider:

- a base vector $\mathbf{x} = (0.4, 0.5, 6.8)$ for which we wish to analyze the global sensitivity with regard to target y_i;
- one "neighboring" (variation) vector $\mathbf{x^{min}} = (0.42, 0.505, 6.766)$ which corresponds to smallest observed value for the target of interest, i.e., it corresponds to y_i^{min};
- another "neighboring" (variation) variation vector $\mathbf{x^{max}} = \mathbf{x} = (0.38, 0.49, 6.834)$ which corresponds to the largest observed value for the target of interest, i.e., it corresponds to y_i^{max};

Based on the above two "neighboring" design vectors, the relative variable deviations are $|0.38 - 0.42|/0.4 = 0.1$ for x_1, $|0.49 - 0.505|/0.5 = 0.03$ for x_2 and $|6.834 - 6.766|/6.8 = 0.01$ for x_3. Since the largest deviation is observed for variable x_1, $x_{spr} = x_1 = 0.4$, $x_{spr}^{max} = 0.42$ and $x_{spr}^{min} = 0.38$.

The extreme target values (y_i^{min} and y_i^{max}) found during the global sensitivity estimation are very useful and can be used to compute two more synthetic metrics:

- the relative loss that can be expected for given target y_i:

$$y_i^{loss} = \frac{(y_i^{max} - y_i)}{|y_i|} \,. \tag{5}$$

- the relative gain that can be expected for given target y_i (a large value of this indicator signals that there is significant potential for local optimization centered around the design parameter vector that is analyzed):

$$y_i^{gain} = \frac{(y_i - y_i^{min})}{|y_i|} \,. \tag{6}$$

VI. Optimization Results and Tolerance-analysis of the Pareto Front

A. Optimization Procedure and Results

In order to optimize the electrical machine topology described in Section III we applied the MagOpt implementation of the surrogate-based optimization process sketched in Fig. 5 using SPEA2 [1] as the base MOEA. Inside SPEA2 we used an equal population and archive size of 100 individuals as this setting has generally delivered good results on similar problem types in the past. The parameterization of SPEA2 was standard (i.e., literature-recommended):

- for the SBX crossover operator [42] we choose a crossover probability of 0.9 and a crossover distribution index of 20;
- for the polynomial mutation operator [43] we choose a a mutation probability of $1/6$ – where 6 is the size of our design variable vector – and a mutation distribution index of 20.

The optimization run was allowed to evaluate 10000 individuals using FE simulations. We also applied a total of 5 surrogate-based optimization blocks: after 1000, 2000, 3000, 4000 and 5000 FE-based evaluations. During each of these surrogate-based cycles, after training the regression models (i.e., MLP-ensembles), we ran SPEA2 with the resulting surrogate-based fitness function for 50 generations (i.e., 5000 individuals). At the end of each surrogate-based cycle, we extracted the best 600 individuals out of the 5000 surrogate-based models, re-evaluated them using FE simulations and added the competitive ones to the archive and population of the FE-based optimization run. All in all, when also considering the surrogate-based designs, each final PN was obtained after considering 32000 possible designs: 10000 FE-validated designs out of which 3000 represent the best individuals from the 25000 surrogate-discovered individuals.

Out of all the individuals discovered during the SPEA2 run, 1004 are Pareto non-dominated when considering the four optimization objectives and thus form the PN. The two-dimensional projections of the corresponding PF is presented in Fig. 7. This projection can be seen a general state-of-the-art result one could expect from successfully running an efficient multi-objective optimization method on a given electrical drive design scenario. In order to speed-up the discovery of this general optimization result, we employed the surrogate-based strategy mentioned in Section IV and introduced in [20] as well as other enhancements presented in [44]. From this point on, the decision maker (electrical engineer) is usually tasked with finding a particular design that exhibits advantageous trade-offs. The decision maker's decision is subjective as it is often influenced by experience and/or personal beliefs. The one point that we wish to highlight in the next sections is that by reusing the already-built surrogates to also perform a very fast sensitivity analyses of the optimized designs, one can provide the decision maker with important extra information regarding expected design robustness at virtually no extra cost. In section VI-C, we show how having this information might, for example, influence a selection process that is primarily focused on cost and efficiency but also aims for a low cogging torque.

B. Design-wise Local and Global Sensitivities

In order to illustrate how the proposed local and global sensitivity estimation strategies work, after the end of the optimization procedure, we applied them on one optimized design selected by the decision maker from the high-efficiency / medium-cost section of the Pareto front. We first performed very fast surrogate-based analyses and then we re-evaluated them using FE simulations. We evaluated a total of 1066 neighboring designs: 66 neighboring designs were required for estimating the local sensitivity and 1000 neighboring designs were required by the global sensitivity part. The very fast surrogate-based analysis based on these 1066 designs took $\approx$ 7 seconds of local computation while the re-evaluation of all the neighboring designs using FE simulations took $\approx$ 6 hours when distributing over 50 HTCondor™ [25] nodes.

The resulting surrogate-based and FE-based local sensitivity maps (matrices) are presented in Fig. 6. Although, the maximum Δ_{loc} value estimated using the ensemble surrogate

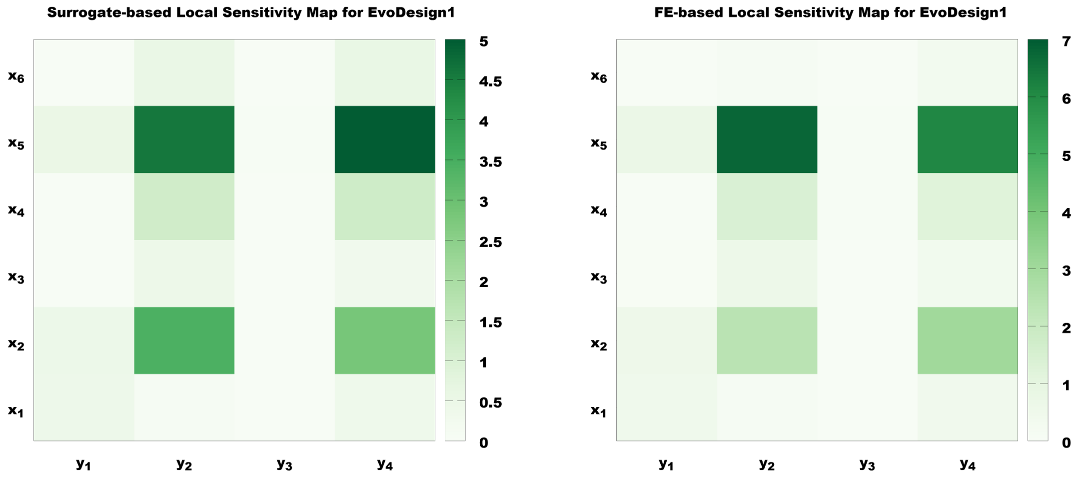

Fig. 6. Surrogate-based and FE-based local sensitivity maps

models is slightly optimistic (≈ 5), the overall structure of the two local sensitivity maps is very similar: two targets are more sensitive ($y_2 = T_{cogpp}$ and $y_4 = T_{rippp}$) and three variables influence these local sensitivities ($x_2 = \alpha_m$, $x_5 = d_{si}$ and, to a lesser extent, $x_4 = e_r$).

The results concerning global sensitivities are displayed in Table IV. The first row in the table shows the training R^2 achieved by each surrogate during 10-fold cross-validation. The magnitudes of both the surrogate and the FE-based results indicate that targets y_2 and y_4 are highly sensitive and target $y_1 = c_{mat}$ is slightly sensitive. Target $y_3 = \eta$ displays virtually no local or global sensitivity. Surrogate and FE-based Δ_{glob} values are similar, but the FE-based analysis estimates losses for y_2 and y_4 that are nearly twice as large. Nevertheless, the surrogate-based global sensitivity analysis seems appropriate for at least making target-related inferences at a broad level – i.e., highly sensitive, sensitive, slightly sensitive, etc. Furthermore, since surrogate-based y_i^{min} and y_i^{max} estimations are quite accurate, this *robustness-related information* can be used to complement the Pareto non-domination *quality-related information* when opting for a particular electrical machine design.

C. Pareto Front Extension

Based on the general multi-objective optimization results we obtained (see Fig. 7), let us consider an academic test case where a decision maker is tasked with finding a relatively cheap (price $\leq$ 5 euros) and relatively highly-efficiency ($\eta \geq 90\%$) design that also exhibits very good operational characteristics – e.g., the lowest cogging torque given the price and efficiency constraints.

Starting from the initial requirement to have a value-for-money design, the decision maker will begin the search (final design selection) process by filtering out all the solutions with a total cost higher than 5 Euros or an efficiency smaller than 90%. The remaining designs are highlighted in all two-dimensional PF projections from Fig. 7. Afterwards, the

TABLE IV
GLOBAL SENSITIVITY INDICATORS FOR THE SELECTED DESIGN

Indicator	Tolerance analysis targets			
	y_1	y_2	y_3	y_4
Surrogate R^2	0.9941	0.9664	0.9938	0.9822
Base value	6.9719	0.1062	-0.9169	0.1104
Surrogate-based y_i^{min}	6.8968	0.0989	-0.9171	0.1002
Surrogate-based y_i^{max}	7.0588	0.1140	-0.9165	0.1189
Surrogate-based y_i^{loss}	1.25%	7.34%	0.00%	7.70%
Surrogate-based Δ_{glob}	1.1863	8.4553	0.0306	8.5650
FE-based y_i^{min}	6.8776	0.0998	-0.9171	0.1034
FE-based y_i^{max}	7.0824	0.1207	-0.9142	0.1254
FE-based y_i^{loss}	1.58%	13.65%	0.03%	13.59%
FE-based Δ_{glob}	1.4688	9.8396	0.1617	9.9678

decision maker is likely to look for a design that also features a good value for the cogging torque. In other words, he can focus his search on the designs that define the left-side contour of the highlighted PF sector from the bottom-left subplot (i.e., efficiency vs. cogging torque) from Fig. 7. There are nine designs of interest and variable and objective-wise details regarding these designs are presented in Table V. In Fig. 8 we illustrate how the assessed sensitivity of these nine designs can affect the (expected) trade-off between cogging torque and efficiency.

The transparent rectangles associated to each Pareto non-dominated point/design from Fig. 8 mark the global surrogate-estimated sensitivity, i.e., they are obtained by plotting the min-max estimates for each of the two targets. For performing each point-wise sensitivity estimation we used the same surrogate models as in the previous section.

Fig. 8 contains compelling evidence that augmenting a basic Pareto plot with sensitivity information is highly worthwhile.

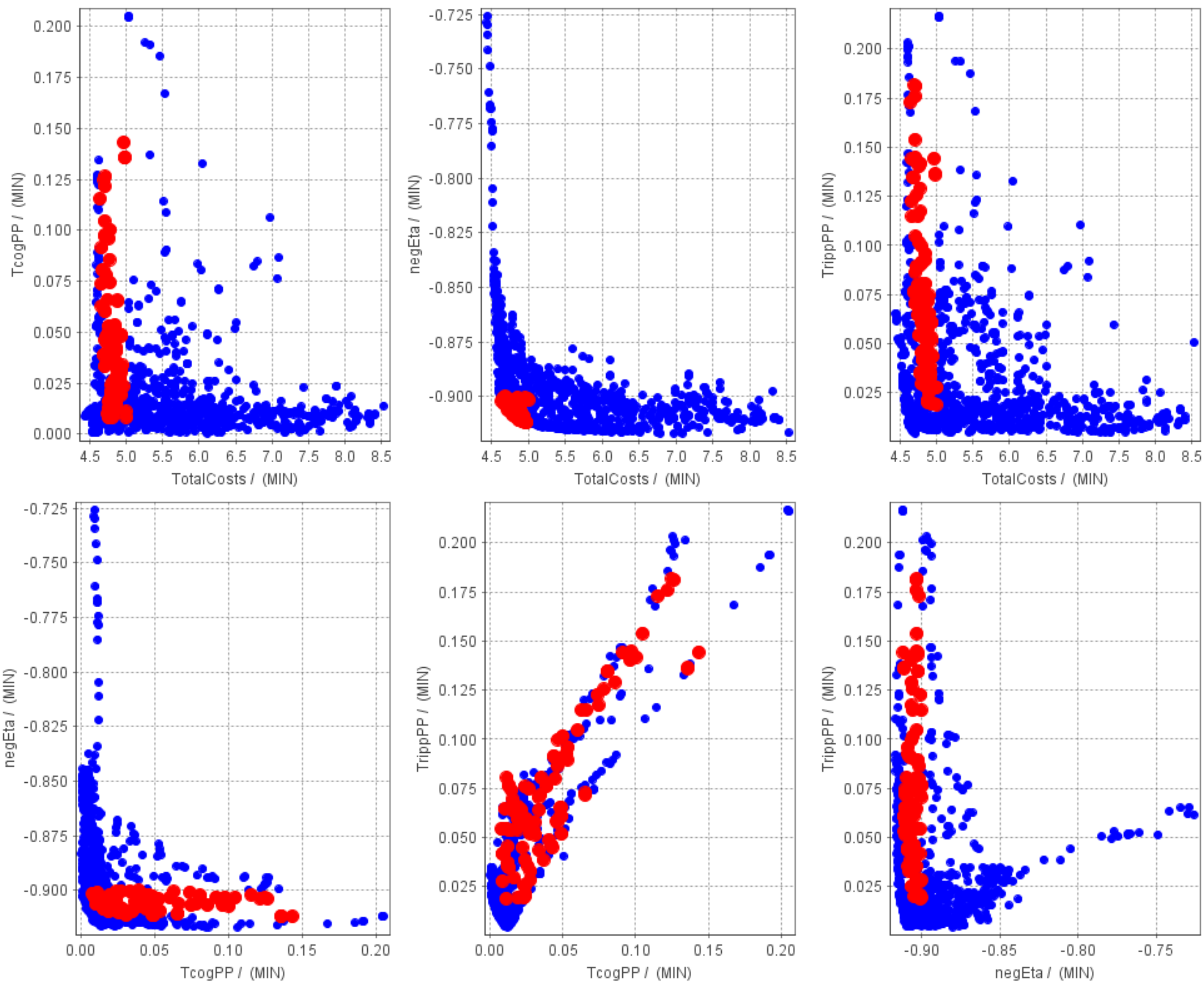

Fig. 7. Two-dimensional objective space projections of the 1004 Pareto non-dominated designs that were discovered after performing the multi-objective optimization phase. The highlighted designs simultaneously have an efficiency higher than 90% and a cost lower than 5 Euros.

First and foremost, it enables one to rapidly identify solutions that are relatively robust (e.g., the designs corresponding to points 4, 5 and 7) or, in contrast, rather sensitive (e.g., the designs corresponding to points 1, 8 and 9). The comparative sensitivity estimates corresponding to neighboring points 1 and 2 (or 7 and 8) show that there is great potential to improve the overall optimization process by treating design robustness as a target (or at least constraint) in its own right.

Secondly, using individual sensitivity values, one can easily create a min-max (worst case - best case) estimation for the entire Pareto front. This Pareto estimation can be interpreted as a rudimentary "*confidence band*" for the general result of the initial multi-objective optimization problem. It is very important to underline that the precision of the Pareto confidence band is highly dependent on the quality of the estimations provided by the surrogate regression models.

Thirdly, one can gain valuable insight into the overlap degree between different solutions (e.g., designs corresponding to points 7 and 8). The overlap degree can be subsequently used (as an objective-space metric) to filter candidate solutions, either during or at the end of the optimization run. Additionally, some designs that are "slightly Pareto-dominated" (i.e., they are not in the first Pareto front) might prove very robust with min-max sensitivity estimates that are completely encompassed by the far larger sensitivities of the Pareto-optimal designs. In this case, a choice for the slightly dominated designs is well-worth considering. In our case, this means that the design corresponding to point 8 would likely be preferred to the one corresponding to point 7 even if the former also exhibited a slightly worse cogging torque measure than the latter (and thus be Pareto-dominated). Likewise, based on robustness with regard to cogging torque estimation, design no. 2 is to be preferred to design no. 1 when aiming for the highest possible efficiency ($\approx 91\%$) among the reduced set of optimized designs.

All in all, by using information provided by the sensitivity analysis strategy that we propose, the decision maker can further filter the number of suitable designs by removing those that appear to be less robust than their neighbors (i.e., designs corresponding to points number 1, 8, and 9 in the current

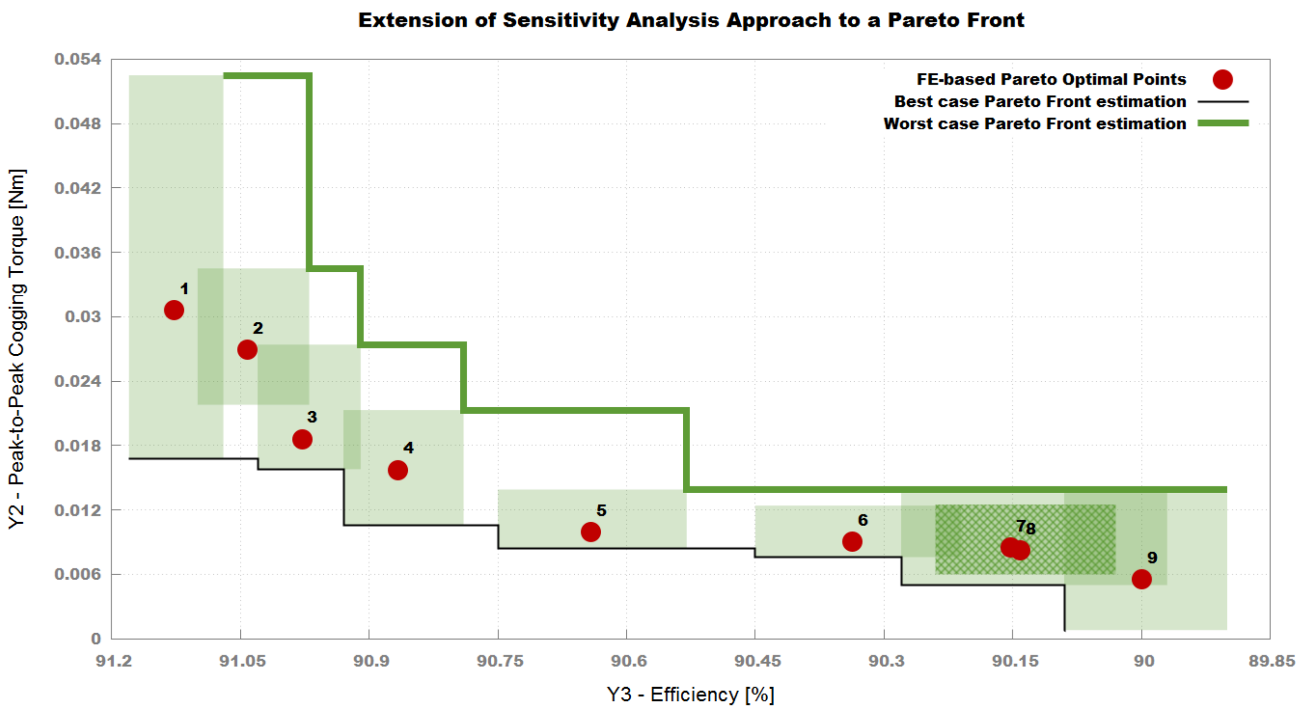

Fig. 8. The extension of our surrogate-based sensitivity analysis approach to an FE-based Pareto front. The transparent min-max sensitivity box associated with point 8 is also hashed in order to better distinguish it from the larger encompassing box associated with point 7.

TABLE V
THE DESIGN PARAMETER VECTORS AND COMPLEMENTARY OBJECTIVE VALUES FOR THE NINE DESIGNS PLOTTED IN FIG. 8.

Design no.	Design variables						Design objectives			
	h_m [mm]	α_m [1]	h_r [mm]	e_r [mm]	d_{si} [mm]	b_{st} [mm]	c_{mat} [Euro]	T_{cogpp} [Nm]	η [%]	T_{rippp} [Nm]
1	1.0	0.90	2.0	0.0	45.0	4.0	4.920	0.0305	91.126	0.0534
2	1.0	0.85	1.9	0.0	46.0	3.7	4.886	0.0269	91.041	0.0747
3	1.0	0.85	1.8	0.0	45.5	4.0	4.874	0.0186	90.978	0.0640
4	1.0	0.80	1.5	0.0	46.0	3.5	4.832	0.0156	90.866	0.0694
5	1.0	0.85	1.9	0.3	45.0	3.8	4.823	0.0098	90.641	0.0416
6	1.0	0.80	1.5	0.3	46.0	4.5	4.810	0.0090	90.336	0.0416
7	1.2	0.80	1.5	0.5	45.0	4.8	5.000	0.0084	90.152	0.0280
8	1.0	0.75	1.0	0.2	45.5	4.6	4.750	0.0081	90.141	0.0542
9	1.0	0.75	0.6	0.3	45.0	3.8	4.754	0.0055	90.005	0.0476

study) and have some degree of confidence that the trade-offs indicated for the other designs will hold in the face of (manufacturing) deviations within pre-established tolerance intervals. Of course, given a real-life design task, one could and should proceed to also look at the sensitivity-related information for torque ripple and total costs for the nine designs.

VII. CONCLUSION

In the present work we have introduced a practical two-stage tolerance analysis strategy based on the min-max approach. The industrial case study shows that global surrogate (i.e., nonlinear regression) models constructed for speeding-up multi-objective optimization runs can be easily reused for the proposed tolerance analysis method. This results in the ability to deliver very insightful (and to a certain degree accurate) estimations of expected target deviations induced by given (local) changes in the inputs of the optimized electrical motor designs.

Extending the new surrogate-based sensitivity analysis approach to the entire Pareto front generated by the multi-objective optimization enables the creation of a rudimentary Pareto confidence band. Analyzing the sensitivity of every solution from the current Pareto-front would take around 20 minutes of local computation when using the surrogate models. The complementary FE-based analysis of all the candidate solutions would be nearly impossible (i.e., last longer than 20 days) even when using a computer cluster given the ≈ 6 hours required for each design. In turn, the Pareto confidence band would enable researchers and practitioners to better frame their expectations with regard to the result of the optimization run. Another view on the matter is that these preliminary tolerance results combined with the ability to produce them very fast using surrogate models give weight to a future research direction that considers the tolerance/sensitivity associated with a certain design as an optimization objective in its own right. This allows for considering the trade-off between rated performance vs. sensitivity. In addition, the search direction of the applied optimization algorithm can be adapted if constraints on maximum permissible sensitivities or on maximum feasible deviations due to tolerances are set.

VIII. Outlook

This work was about possibilities for incorporating tolerance or sensitivity analyses to typical multi-objective optimization scenarios considered for electric machine design. The sensitiveness of the objectives of the optimization problem with regard to all design parameters was analyzed. In order to speed up the evaluation, surrogate models were used that were initially derived for speeding up the optimization process.

Engineers dealing with electric machine design need to consider further tolerances associated with parameters that usually are not subject of optimization. For instance, a static or dynamic eccentricity of the rotor, unequal stator tooth widths, or permanent magnets with different material properties need to be taken into account. If those nonidealities need to be analyzed during optimization, computational effort for evaluating designs will increase dramatically.

Investigating the sensitivity of Pareto optimal points after completing an optimization run is one possible option in order to minimize the computational effort. However, if the sensitivity is not already considered during optimization, this might cause the optimizer to steer the search to more sensitive regions where good rated performance can be obtained. Thus, focus should be on deriving additional surrogate models or equivalent techniques to facilitate considering the reliability of designs under consideration as early as possible.

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References

[1] E. Zitzler, M. Laumanns, and L. Thiele, "SPEA2: Improving the Strength Pareto Evolutionary Algorithm for Multiobjective Optimization," in *Evolutionary Methods for Design, Optimisation and Control with Application to Industrial Problems (EUROGEN 2001)*, 2002, pp. 95–100.

[2] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan, "A fast and elitist multiobjective genetic algorithm: NSGA-II," *IEEE Transactions on Evolutionary Computation*, vol. 6, no. 2, pp. 182–197, 2002.

[3] Y. del Valle, G. Venayagamoorthy, S. Mohagheghi, J.-C. Hernandez, and R. Harley, "Particle swarm optimization: Basic concepts, variants and applications in power systems," *IEEE Transactions on Evolutionary Computation*, vol. 12, no. 2, pp. 171–195, April 2008.

[4] K. Price, "Differential evolution: a fast and simple numerical optimizer," in *1996 Biennial Conference of the North American Fuzzy Information Processing Society, 1996. NAFIPS.*, Jun 1996, pp. 524–527.

[5] A.-C. Zăvoianu, G. Bramerdorfer, E. Lughofer, W. Amrhein, and E. P. Klement, "DECMO2: A Robust Hybrid and Adaptive Multi-Objective Evolutionary Algorithm," *Soft Computing*, 2014.

[6] A.-C. Zăvoianu, E. Lughofer, W. Koppelstätter, G. Weidenholzer, W. Amrhein, and E. P. Klement, "On the performance of master-slave parallelization methods for multi-objective evolutionary algorithms," in *Proceedings of the 12th International Conference on Artificial Intelligence and Soft Computing - ICAISC 2013*. Springer Berlin / Heidelberg, 2013.

[7] G. Bramerdorfer, S. Silber, G. Weidenholzer, and W. Amrhein, "Comprehensive cost optimization study of high-efficiency brushless synchronous machines," in *2013 IEEE International Electric Machines and Drives Conference (IEMDC)*, 2013, pp. 1126–1131.

[8] C.-C. Chiang, M.-F. Hsieh, Y.-H. Li, and M.-C. Tsai, "Impact of electrical steel punching process on the performance of switched reluctance motors," *IEEE Transactions on Magnetics*, vol. 51, no. 11, pp. 1–4, Nov 2015.

[9] S.-K. Kuo, W.-C. Lee, S.-Y. Lin, and C.-Y. Lu, "The influence of cutting edge deformations on magnetic performance degradation of electrical steel," *IEEE Transactions on Industry Applications*, vol. 51, no. 6, pp. 4357–4363, Nov 2015.

[10] O. Hubert and S. Lazreg, "Multidomain modeling of the influence of plastic deformation on the magnetic behavior," *IEEE Transactions on Magnetics*, vol. 48, no. 4, pp. 1277–1280, April 2012.

[11] M. Centner and U. Schafer, "Optimized design of high-speed induction motors in respect of the electrical steel grade," *IEEE Transactions on Industrial Electronics*, vol. 57, no. 1, pp. 288–295, Jan 2010.

[12] N. Bianchi, M. Degano, and E. Fornasiero, "Sensitivity analysis of torque ripple reduction of synchronous reluctance and interior pm motors," *IEEE Transactions on Industry Applications*, vol. 51, no. 1, pp. 187–195, Jan 2015.

[13] M. Chowdhury, M. Islam, A. Gebregergis, and T. Sebastian, "Robust design optimization of permanent magnet synchronous machine utilizing genetic and taguchi's algorithm," in *2013 IEEE Energy Conversion Congress and Exposition (ECCE)*, Sept 2013, pp. 5006–5012.

[14] Z. Ren, M.-T. Pham, and C. S. Koh, "Robust global optimization of electromagnetic devices with uncertain design parameters: Comparison of the worst case optimization methods and multiobjective optimization approach using gradient index," *IEEE Transactions on Magnetics*, vol. 49, no. 2, pp. 851–859, Feb 2013.

[15] G. Lei, Y. Guo, J. Zhu, T. Wang, X. Chen, and K. Shao, "System level six sigma robust optimization of a drive system with pm transverse flux machine," *IEEE Transactions on Magnetics*, vol. 48, no. 2, pp. 923–926, Feb 2012.

[16] G. Lei, T. Wang, J. Zhu, Y. Guo, and S. Wang, "System-level design optimization method for electrical drive systems: Robust approach," *IEEE Transactions on Industrial Electronics*, vol. 62, no. 8, pp. 4702–4713, Aug 2015.

[17] T.-S. Low, S. Chen, and X. Gao, "Robust torque optimization for bldc spindle motors," *IEEE Transactions on Industrial Electronics*, vol. 48, no. 3, pp. 656–663, Jun 2001.

[18] J. Jang, S. gil Cho, S.-J. Lee, K.-S. Kim, J.-M. Kim, J.-P. Hong, and T. H. Lee, "Reliability-based robust design optimization with kernel density estimation for electric power steering motor considering manufacturing uncertainties," *IEEE Transactions on Magnetics*, vol. 51, no. 3, pp. 1–4, March 2015.

[19] S. Lee, K. Kim, S. Cho, J. Jang, T. Lee, and J. Hong, "Optimal design of interior permanent magnet synchronous motor considering the manufacturing tolerances using taguchi robust design," *IET Electric Power Applications*, vol. 8, no. 1, pp. 23–28, January 2014.

[20] A.-C. Zăvoianu, G. Bramerdorfer, E. Lughofer, S. Silber, W. Amrhein, and E. P. Klement, "Hybridization of multi-objective evolutionary algorithms and artificial neural networks for optimizing the performance of electrical drives," *Engineering Applications of Artificial Intelligence*, vol. 26, no. 8, pp. 1781–1794, 2013.

[21] A.-C. Zăvoianu, E. Lughofer, G. Bramerdorfer, W. Amrhein, and E. P. Klement, "An effective ensemble-based method for creating on-the-fly

surrogate fitness functions for multi-objective evolutionary algorithms," in *SYNASC 2013 - 15th International Symposium on Symbolic and Numeric Algorithms for Scientific Computing*, 2013.

[22] S. Giurgea, H. Zire, and A. Miraoui, "Two-stage surrogate model for finite-element-based optimization of permanent-magnet synchronous motor," *IEEE Transactions on Magnetics*, vol. 43, no. 9, pp. 3607–3613, Sept 2007.

[23] G. Tsekouras, S. Kiartzis, A. Kladas, and J. Tegopoulos, "Neural network approach compared to sensitivity analysis based on finite element technique for optimization of permanent magnet generators," *IEEE Transactions on Magnetics*, vol. 37, no. 5, pp. 3618–3621, Sep 2001.

[24] J. Sykulski, A. Al-Khoury, and K. Goddard, "Minimal function calls approach with on-line learning and dynamic weighting for computationally intensive design optimization," *IEEE Transactions on Magnetics*, vol. 37, no. 5, pp. 3423–3426, Sep 2001.

[25] D. Thain, T. Tannenbaum, and M. Livny, "Distributed computing in practice: the condor experience." *Concurrency - Practice and Experience*, vol. 17, no. 2-4, pp. 323–356, 2005.

[26] F. Poltschak and W. Amrhein, "A dynamic nonlinear model for permanent magnet synchronous machines," in *IEEE International Symposium on Industrial Electronics, 2008. ISIE 2008.*, June 2008, pp. 724–729.

[27] G. Bramerdorfer, S. Winkler, M. Kommenda, G. Weidenholzer, S. Silber, G. Kronberger, M. Affenzeller, and W. Amrhein, "Using FE calculations and data-based system identification techniques to model the nonlinear behavior of PMSMs," *IEEE Transactions on Industrial Electronics*, vol. 61, no. 11, pp. 6454–6462, Nov. 2014.

[28] G. Weidenholzer, S. Silber, G. Jungmayr, G. Bramerdorfer, H. Grabner, and W. Amrhein, "A flux-based PMSM motor model using RBF interpolation for time-stepping simulations," in *2013 IEEE International Electric Machines & Drives Conference (IEMDC)*, 2013, pp. 1418–1423.

[29] G. Sizov, D. Ionel, and N. Demerdash, "A review of efficient FE modeling techniques with applications to PM AC machines," in *2011 IEEE Power and Energy Society General Meeting*, July 2011, pp. 1–6.

[30] G. Y. Sizov, P. Zhang, D. M. Ionel, N. A. O. Demerdash, and M. Rosu, "Automated multi-objective design optimization of PM AC machines using computationally efficient FEA and differential evolution," *IEEE Transactions on Industry Applications*, vol. 49, no. 5, pp. 2086–2096, Sept. 2013.

[31] G. Sizov, D. Ionel, and N. Demerdash, "Modeling and parametric design of permanent-magnet ac machines using computationally efficient finite-element analysis," *IEEE Transactions on Industrial Electronics*, vol. 59, no. 6, pp. 2403–2413, June 2012.

[32] G. Y. Sizov, D. M. Ionel, and N. A. O. Demerdash, "Multi-objective optimization of PM AC machines using computationally efficient - FEA and differential evolution," in *2011 IEEE International Electric Machines and Drives Conference (IEMDC)*, May 2011, pp. 1528–1533.

[33] C. Coello Coello, G. Lamont, and D. Van Veldhuisen, *Evolutionary Algorithms for Solving Multi-Objective Problems*, ser. Genetic and Evolutionary Computation Series. Springer, 2007.

[34] S. Silber, W. Koppelstätter, G. Weidenholzer, and G. Bramerdorfer, "MagOpt - Optimization Tool for Mechatronic Components," in *14th International Symposium on Magnetic Bearings (ISMB14)*, 2014.

[35] S. Silber, G. Bramerdorfer, A. Dorninger, A. Fohler, J. Gerstmayr, W. Koppelstätter, D. Reischl, G. Weidenholzer, and S. Weitzhofer, "Coupled optimization in MagOpt," *Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering*, 2015.

[36] A.-C. Zăvoianu, G. Bramerdorfer, E. Lughofer, S. Silber, W. Amrhein, and E. P. Klement, "A hybrid soft computing approach for optimizing design parameters of electrical drives," in *Soft Computing Models in Industrial and Environmental Applications*, 2013, vol. 188, pp. 347–358.

[37] Linz Center of Mechatronics GmbH (LCM), Website http://www.lcm.at.

[38] S. Haykin, *Neural Networks: A Comprehensive Foundation (2nd Edition)*. Upper Saddle River, New Jersey: Prentice Hall Inc., 1999.

[39] C. M. Creveling, *Tolerance design: a handbook for developing optimal specifications*. Prentice Hall, 1997.

[40] C. Wu and M. Hamada, *Experiments: planning, design, and parameter optimization*. Wiley, New York, 2000.

[41] M. D. McKay, R. J. Beckman, and W. J. Conover, "Comparison of three methods for selecting values of input variables in the analysis of output from a computer code," *Technometrics*, vol. 21, no. 2, pp. 239–245, 1979.

[42] K. Deb and R. B. Agrawal, "Simulated binary crossover for continuous search space," *Complex Systems*, vol. 9, pp. 115–148, 1995.

[43] K. Deb and M. Goyal, "A combined genetic adaptive search (GeneAS) for engineering design," *Computer Science and Informatics*, vol. 26, pp. 30–45, 1996.

[44] G. Bramerdorfer, A.-C. Zavoianu, S. Silber, E. Lughofer, and W. Amrhein, "Possibilities for speeding up the fe-based optimization of electrical machines - a case study," *IEEE Transactions on Industry Applications*, vol. 52, no. 6, pp. 4668 – 4677, 2016.

Gerd Bramerdorfer (S'10-M'14) received the PhD in Electrical Engineering in 2014 from Johannes Kepler University Linz, Austria, and he is currently an Assistant Professor with the same university.

Since 2007 he has been with the Department of Electrical Drives and Power Electronics, Johannes Kepler University Linz, where he was involved in various research projects and held classes in the field of electric machines and power electronics. His research interests comprise the design, modeling, and optimization of high-efficient brushless electric machines as well as magnetic bearings and bearingless machines.

He is a member of IEEE, the IEEE Industry Applications Society (IAS), the IEEE Industrial Electronics Society (IES), the IES Electrical Machines Technical Committee and a reviewer for several IEEE journals and conferences.

Alexandru-Ciprian Zăvoianu received the PhD in Computer Science in 2015 from the Johannes Kepler University Linz, Linz, Austria and is currently a postdoctoral researcher at the Department of Knowledge-Based Mathematical Systems at the same university. His research interests include stochastic optimization, evolutionary computation, machine learning, data mining and distributed computing. He is a member of the IEEE Technical Committee on Soft Computing (IEEE SMC) and a reviewer for several journals and conferences in the field.