Multi-Objective Topology Optimization of Electrical Machine Designs using Evolutionary Algorithms with Discrete and Real Encodings

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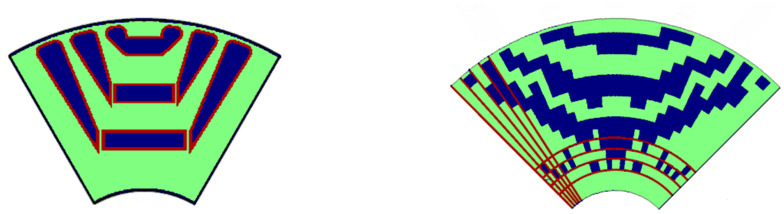
Abstract. We describe initial results obtained when applying different multi-objective evolutionary algorithms (MOEAs) to direct topology optimization (DTO) scenarios that are relevant in the field of electrical machine design. Our analysis is particularly concerned with investigating if the use of discrete or real-value encodings combined with a preference for a particular population initialization strategy can have a severe impact on the performance of MOEAs applied for DTO.

Keywords: evolutionary algorithms, multi-objective optimization, discrete encoding, real encoding, topology optimization, electrical machine design

1 Introduction and Motivation

When designing electrical motors, one generally aims to discover machines that are simultaneously optimal with regard to (at least a few of) several criteria like energy efficiency, manufacturing costs, fault tolerance and operating characteristics. The standard approach for tackling these real-life multi-objective optimization problems (MOOPs) is structured as a two-step procedure. In the first step, a domain expert (i.e., an electrical engineer) defines the complete geometric specifications of the future design. This actually means that the human expert creates or chooses (and likely adapts) a parametric model that will act as a generic template for any subsequent electrical drive design that aims to solve the given task (see Figure 1a). In the second step, a multi-objective optimization algorithm (MOOA) is employed to discover those sets of parameter combinations that, when applied to the preselected generic template, will produce Pareto-optimal design solutions. The final choice for one (or more) of the Pareto-optimal designs rests with the domain expert or with a third party decision maker (i.e., a customer).

Even when carrying out the optimization part via state-of-the-art population-based meta-heuristic solvers, like hybrid multi-objective evolutionary algorithms



(a) Based on parametric model (b) Based on discrete grid

Fig. 1: Example of two cross-sections of rotor designs. Light green parts denote iron elements and dark blue parts denote air gaps.

(MOEAs) [12] and particle swarm optimization strategies [1], one can easily argue that the truly creative part of the design process remains with the domain expert. When the wrong parametric model for the task at hand is chosen, no amount of optimization will be able to deliver good results. Thus, by imposing hard constraints in variable space, a choice for a parametric model actually entails restrictions on the shape of possible designs.

Direct topology optimization (DTO) [5] is an alternative approach that, when applicable, seems better suited to fully benefit from the explorative strength of modern MOEAs and recent advances in simulation software and computation power [7]. In this case, the domain expert only needs to define the boundaries of the design region and to choose a discretization factor. This results in a grid in which each cell can be parameterized from a limited set of values (see Figure 1b). The simplest of such sets contains only two elements: iron and air. The task of the MOOA is to find those grid configurations (i.e., discrete matrices) that encode Pareto-optimal solutions. Thus, since the optimization problem is formulated in a manner that imposes virtually no restrictions on attainable geometries, the MOOA also "becomes responsible" for the more advanced / innovative part of the design automation process.

2 Research Focus and Approach

Our current aim is to gain some insights regarding the expected performance of multi-objective evolutionary algorithms (MOEAs) used on direct topology optimization scenarios. As such, we performed different types of numerical experiments on artificial and industrial problems. In particular, we investigated if the MOEAs currently used for template-based multi-objective optimization scenarios [8] are also suitable for DTO after minimal modifications. Across all DTO experiments, we have chosen to concatenate the rows of the topology matrix and to use a uni-dimensional (i.e., vector) encoding for all the tested MOEAs.

Our first idea was to adapt the very well known NSGA-II [3] to a DTO context by simply fitting it with genetic operators suitable for a *discrete-value encoding*: single point crossover and bit-flip mutation. This discrete encoding is the natural codification for a topology matrix / vector as a value of 0 can be used to denote air (i.e., cavities in the rotor design) and increasing non-negative values can encode various construction materials (e.g., iron, copper, magnets with different magnetization directions, etc.). In the present work we focus on the simplest (binary) DTO scenarios that consider only iron and air elements.

Secondly, we tested if the good convergence behavior exhibited by more advanced hybrid MOEAs – like DECMO [14] and DECMO2 [15] – on template-based scenarios also translates to DTO. Since a key feature of the two hybrid MOEAs is the integration of differential evolution (DE) [9] – a continuous optimization paradigm – applying them to DTO scenarios also requires a rather counter-intuitive *real-value encoding* of the topology matrices / vectors.

In order to get a better overview of the MOEA performance, we also tested different initialization strategies for both types of encodings. Thus, at the individual (i.e., topology vector) level we considered four initialization options: *1* – all cells are initialized with the value 1 (iron), *0* – all cells are initialized with the value 0 (air), B – each cell is initialized randomly with either 0 or 1, R – each cell is initialized randomly with a uniformly sampled value from $[0, 1]$. The R option is specific to the real-value encoding.

At the (start) population level we considered 6 initialization options. In four of them (marked by the prefix "all-") every member of the population was initialized using the same individual strategy: *all-1*, *all-0*, *all-B*, *all-R*. In the case of two population initialization strategies, namely *0/1/B* and *0/1/R*, the overall start population of the MOEA was divided into three subgroups of equal size and each subgroup was initialized with a different strategy.

3 Experimental Setup

Multi-Objective Solvers

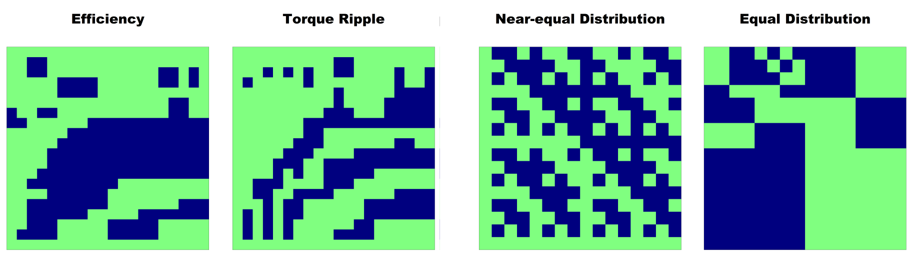
NSGA-II [3] is probably the best known metaheuristic multi-objective optimization strategy and can now be regarded as a (classical) go-to MOEA. Its main feature is a two-tier selection for survival operator based on a primary non-dominated sorting criterion and on a secondary objective-space crowding distance (i.e., niching) quality discriminant. NSGA-II also popularized the usage of two genetic operators: simulated binary crossover and polynomial mutation [2].

DECMO [14] is a proof-of-concept hybrid MOEA based on cooperative co-evolution that uses two subpopulations of equal size in order to integrate two different search strategies during a single optimization run. Thus, one subpopulation relies on an evolutionary model that is similar to the one in NSGA-II while the other subpopulation is evolved via a differential evolution strategy resembling the one proposed in GDE3 [6].

DECMO2 [15] is an improvement over its coevolutionary predecessor as the former was specially designed for rapid convergence on a wide class of problems. The main novelty with respect to DECMO is the integration of an external archive of elite solutions that is maintained according to a decomposition-based principle similar to the one proposed by MOEA/D [11]. In order to smoothly accommodate all three multi-objective search space exploration paradigms, the fitness sharing mechanism in DECMO2 has also been redesigned to dynamically pivot towards the best performing strategy by allowing the latter to generate more offspring during certain parts of the optimization run.

Test Problems and Fitness Assessment

Even though various efficiency-related enhancements have enabled MOEAs to become state-of-the-art solvers in electrical machine design [13], a single optimization run can still take a few days even when distributing the computations over a high-throughput computing cluster of 50-100 nodes. Since MOEAs are stochastic methods, several repeats of an optimization experiment are required in order to estimate the average performance of these solvers. In light of these considerations, for this preliminary study of MOEA performance on DTOs, we have chosen to conduct extensive numerical experiments on two (self-defined) artificial benchmark problems. In the last part of Section 4 we also present results obtained by applying a MOEA to a realistic DTO scenario.



(a) Problem no. 1 – realistic design. (b) Problem no. 2 – artificial pattern.

Fig. 2: Artificial benchmark problems.

Each of the two artificial DTO benchmark MOOPs contains two objectives and each objective is defined as a binary template matrix (see Figure 2). The idea is that, ideally, at the end of an optimization run, the solutions discovered by the MOEA at one extremum of the Pareto front should resemble the first template matrix while the solutions at the other extremum should resemble the second one. In-between Pareto non-dominated solutions are expected to: (1) contain all the subsections that are common in both templates and (2) cover the various trade-offs between the two objective matrices.

The binary (air and iron) templates that define the first benchmark DTO are shown in Figure 2a and their are based on two realistic rotor design patterns. The templates have a size of 20×20 elements (yielding a binary vector encoding of size 400) and, when considering both of them, the air (dark blue) to total elements ratio is 38.25%. The second benchmark DTO MOOP is defined by the

16×16 binary templates from Figure 2b. The artificial patterns that define the second benchmark problem are more balanced (combined air to total elements ratio of 48.44%) but they describe fundamentally different designs (checkered vs continuous).

For both discrete and real value encodings, the internal MOEA fitness of an individual design vector $\mathbf{x}$ of size n with regard to the binary objective template $\mathbf{p}$ was computed as: $f(\mathbf{x}) = \sum_{0\leq i<n} |x_i - p_i|$. In the case of the real value encodings, the presented results were recalculated / post-processed via a threshold-based modification: $f_R(\mathbf{x}) = \sum_{0\leq i<n} |Round(x_i) - p_i|$.

MOEA Parameterization and Assessment of Solution Quality

For the numerical experiments on the two artificial MOOPs, we used the literature recommended genetic operators and standard parameter settings for all 3 MOEAs. The (total) population size was set at 200 and each optimization run was stopped after 100,000 fitness evaluations (i.e., 500 generations).

In order to estimate the convergence performance of MOEAs, we repeated each optimization run 50 times and, at every generation, we recorded the average relative hypervolume [4] of the the MOEA population. The choice for the hypervolume metric is motivated by the monotonic behavior of this unary multi-objective quality indicator and the theoretical relative upper bound of 1 which is especially useful for MOOPs that, like our 2 benchmark problems, have an unknown Pareto-optimal set (i.e., solution).

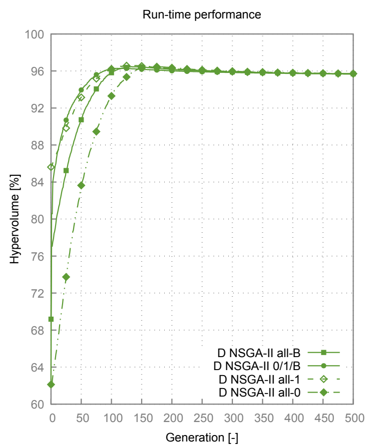
4 Results - Comparative Performance

In Figure 3 we present the comparative convergence performance of NSGA-II with four different initialization options suitable for discrete encodings. The difference from Figure 3a between the *all-0* and *all-1* NSGA-II variants indicates that coupling the initialization strategy with the expected imbalance between air and iron elements can increase the overall convergence speed of the MOEA. The *0/1/B* initialization strategy delivers robust performance on both test MOOPs.

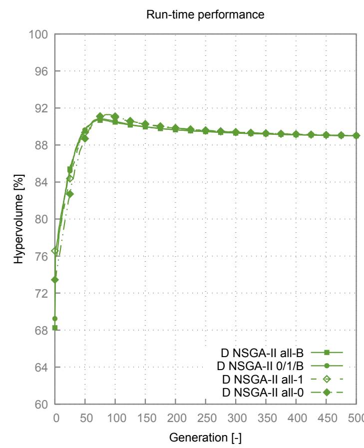
Figure 4 contains the convergence performance of real-value variants of NSGA-II, DECMO and DECMO2 for *all-R* and *0/1/R* – the best performing initialization strategies for real-value encodings. The results of the best discrete NSGA-II variant (*D NSGA-II 0/1/B*) are plotted to ease the comparison. As a general observation, real-value MOEA variants tend to converge slower than their discrete counterparts in the early parts of the run. Nevertheless, the real-value encoding seems to enable MOEAs to maintain a better population diversity and this directly translates into better results towards the end of the optimization.

Among the real-value solvers, DECMO2 coupled with an *all-R* intialization strategy delivers rather good results that are comparable to the ones obtained by the fastest converging discrete variants in the early stages of the run.

Although the performance of DECMO seems to be slightly inferior even to that of the real-value variants of NSGA-II, the former solver obtained very good

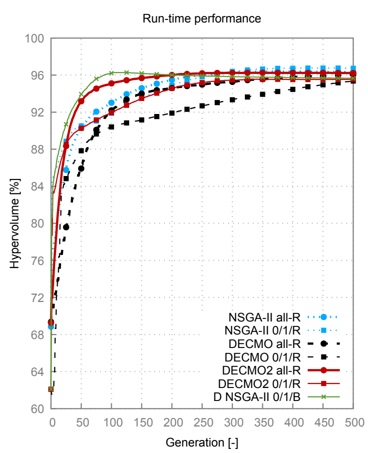


(a) Problem no. 1.

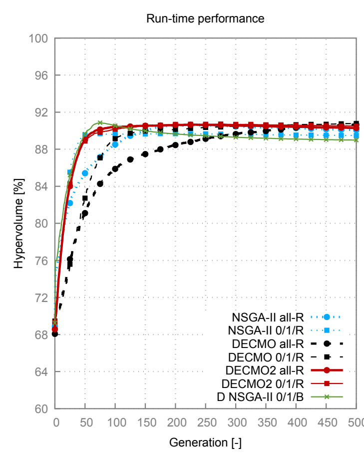


(b) Problem no. 2.

Fig. 3: Performance of NSGA-II with discrete encodings and different initialization strategies.



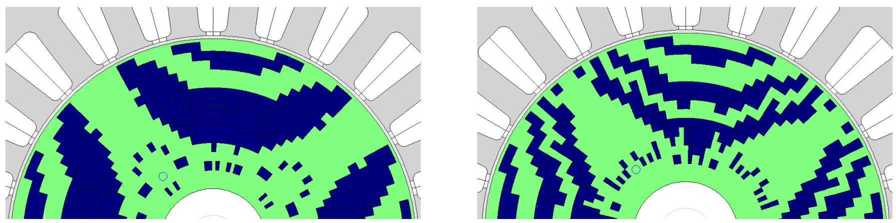
(a) Problem no. 1.



(b) Problem no. 2.

Fig. 4: Comparative convergence performance of the three MOEAs with real encodings versus the best performing version of NSGA-II with discrete encodings.

preliminary results in a realistic DTO scenario [10] in which fitness assessment was performed via finite element (FE) simulations. This realistic multi-objective DTO task is related to a rotor design where one wises to simultaneously optimize output power and torque ripple. The design template is binary, as it allows for only stainless steel and air elements, and has a size of 15×15 (when considering symmetries). Figure 5 contains two rotor designs selected from the extrema of a Pareto front that was obtained after letting DECMO *all-R* evolve 20,000 designs (i.e., run for 100 generations).



(a) Design that maximizes output power. (b) Design that minimizes torque ripple.

Fig. 5: Rotor designs obtained by DECMO (with a real-value encoding) for the realistic DTO scenario.

5 Conclusions and Future Work

The results obtained on artificial and realistic multi-objective optimization problems indicate that applying existing MOEAs with discrete and real-value encodings to direct topology optimization (DTO) scenarios in the filed of electrical machine design can yield very promising solutions. As the preliminary innovative designs delivered by coupling MOEAs and DTO have been validated by domain experts (i.e., electrical engineers), the present study can be seen as a first step towards a more symbiotic relation between human experts and automated global search strategies inside the product design cycle.

Future work will revolve around two issues. Firstly, we plan to apply MOEAs to multi-material DTO problems. Secondly, we would like to perform the very computationally-intensive (but extremely useful) comparison between the advantages of using discrete vs. real encodings on multiple FE-based DTO scenarios.

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